Performance of an Adaptive Algorithm for Sinusoidal Disturbance Rejection in High Noise

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Abstract

The paper proposes a discrete-time algorithm for the rejection of sinusoidal disturbances of unknown frequency. The algorithm is adapted from an existing continuous-time algorithm and the overall nonlinear system is analyzed using a linear approximation. The paper shows that the noise rejection properties may be predicted using an in-phase/quadrature decomposition of the noise similar to the one encountered in the theory of phase-locked loops. Estimates are obtained for the standard deviations of the plant output and of the adaptive parameters and simulations validate the predictions of the analysis, despite the nonlinear nature of the adaptive system and the high level of noise applied.

1 Introduction

The rejection of periodic disturbances is a common problem in control applications. If the frequency of the disturbance is known, several techniques are available, including internal model control, adaptive methods, and repetitive control techniques. These methods are closely connected. In particular, it was shown in [1] that a standard adaptive feedforward control algorithm was equivalent to an internal model control law. Examples of applications include helicopters [2] and high-density magnetic disk drives [3].

When the frequency of the disturbance is not known, the problem is considerably more complicated. Sometimes, the disturbance (or a signal related to it) can be measured, and the problem can be solved using an adaptive feedforward control algorithm. However, in many applications, it is inconvenient, if not impossible, to place a sensor on the source of the disturbance or along its path to the plant. For such problems, an adaptive implementation of the internal model principle may be considered [4],[5], [6]. Such algorithms are highly nonlinear and, aside from Lyapunov-type stability results, little is known about their convergence properties or their sensitivity to measurement noise.

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In [7], two adaptive algorithms were proposed for the cancellation of sinusoidal disturbances with unknown frequency. The first algorithm was called *indirect*, and was obtained by combining a frequency estimation algorithm together with an adaptive algorithm for the cancellation of disturbances of known frequency. A comparable approach was followed in [8], using a different frequency estimation algorithm and a repetitive control method. The second algorithm of [7] was called *direct*, and estimated the frequency, phase, and magnitude of the disturbance in an integrated fashion. The scheme was obtained by combining a phase-locked loop concept with a sinusoidal disturbance cancellation scheme. The algorithm was extended to handle multiple sinusoidal components and implemented successfully in an active noise control testbed [9].

As other algorithms for the rejection of periodic disturbances of unknown frequency, the direct algorithm of [7] is highly nonlinear. However, it may be approximated by a linear system using techniques similar to those used for phase-locked loops [10]. As a result, estimates of convergence rates can be obtained that are useful for design. The contribution of this paper is to show that the continuous-time algorithm of [7] may be adapted to discrete-time, and that its noise properties may be predicted using an in-phase/quadrature decomposition of the measurement noise signal. Simulations show that the results of the analysis are accurate, even in the presence of high noise.

2 A Discrete-Time Algorithm

2.1 Problem Statement

The plant is assumed to be described, in the z-transform domain, by

$$\begin{aligned} y(z) &= P(z)(u(z) - d(z)) \\ \bar{y}(z) &= y(z) + n(z) \end{aligned} \tag{1}$$

where u is the control input, d is the disturbance, n is the measurement noise, y is the plant output, and $\bar{y}$ is the measured plant output. $P(z)$ is the transfer function of the plant, which is assumed to be stable. In the time-domain, the disturbance is assumed to be given by

$$d(k) = d_1 \cos(\alpha_d(k)), \tag{2}$$

with

$$\alpha_d(k) = \omega_1 k + \delta_1. \tag{3}$$

The disturbance is a sinusoidal function with magnitude d_1, frequency ω_1, and initial phase δ_1. These parameters are fixed, but unknown. Alternatively, (3) can be written in a recursive form as

$$\alpha_d(k+1) = \alpha_d(k) + \omega_1, \quad \alpha_d(0) = \delta_1. \tag{4}$$

Although the disturbance does not need to act at the input of the plant, it is assumed that an equivalent input disturbance may be so defined. For its cancellation, the input is then chosen to be of the form

$$u(k) = \theta_1(k) \cos(\alpha(k)), \tag{5}$$

with

$$\alpha(k+1) = \alpha(k) + \theta_2(k), \quad \alpha(0) = 0. \tag{6}$$

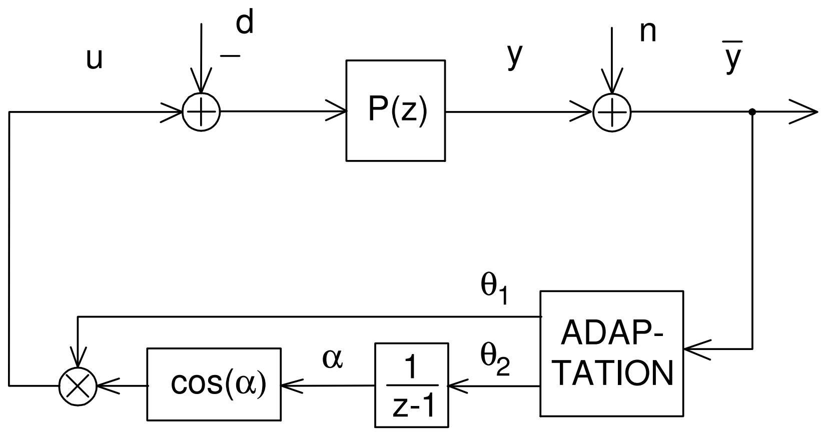

Figure 1: Plant and Adaptive System

The variables $\theta_1(k)$ and $\theta_2(k)$ are parameters with nominal values ${\theta_1}^* = d_1$ and ${\theta_2}^* = \omega_1$. The phase of the disturbance is estimated through the variable $\alpha(k)$, and it is determined by the time history of $\theta_2(k)$ rather than through separate adaptation: even though the problem is stated in terms of three unknown parameters, the solution is based on the adaptation of only two parameters $\theta_1(k)$ and $\theta_2(k)$. Specifically, for the exponentially stable adaptive scheme developed in this paper, the phase δ_1 is determined through

$$\delta_1 = \sum_{k=0}^{\infty}(\theta_2(k) - \omega_1) \tag{7}$$

The structure of the overall system is shown in Fig. 1.

2.2 Basic Results

In a first step, we assume that the measurement noise $n = 0$, so that $y = \bar{y}$. The algorithm is based on the following fact.
Fact:
Assume that:

- θ_1 and θ_2 vary sufficiently slowly that the response of the plant to the signal u can be approximated by the steady-state output of the plant for a sinusoidal input with frequency θ_2.
- the instantaneous frequency θ_2 is close to ω_1, so that $P(e^{j\theta_2})$ can be replaced by $P(e^{j\omega_1})$.
- the phase error $\alpha - \alpha_d$ is small.

Then: considering low-frequency components only, the two signals

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} y(k)\cos(\alpha(k)) \\ -y(k)\sin(\alpha(k)) \end{bmatrix} \tag{8}$$

are approximately given by

$$\left[\begin{array}{c} y_1(k) \\ y_2(k) \end{array}\right] = G \left[\begin{array}{c} \theta_1(k) - d_1 \\ d_1(\alpha(k) - \alpha_d(k)) \end{array}\right], \tag{9}$$

where

$$G = \frac{1}{2}\left[\begin{array}{cc} Re[P(e^{j\omega_1})] & -Im[P(e^{j\omega_1})] \\ Im[P(e^{j\omega_1})] & Re[P(e^{j\omega_1})] \end{array}\right].$$

Proof: Let

$$P_R = Re[P(e^{j\omega_1})], \quad P_I = Im[P(e^{j\omega_1})]. \tag{10}$$

Under the assumptions, the output of the plant is given by

$$\begin{aligned} y(k) &= P_R\theta_1(k)\cos(\alpha(k)) - P_I\theta_1(k)\sin(\alpha(k)) \\ &- P_R d_1\cos(\alpha_d(k)) + P_I d_1 \sin(\alpha_d(k)). \end{aligned} \tag{11}$$

Keeping only the low-frequency components of the signals y_1 and y_2, we have

$$\begin{aligned} y_1(k) &= \frac{1}{2}P_R\theta_1(k) - \frac{1}{2}P_R d_1\cos(\alpha(k) - \alpha_d(k)) \\ &- \frac{1}{2}P_I d_1\sin(\alpha(k) - \alpha_d(k)), \\ y_2(k) &= \frac{1}{2}P_I\theta_1(k) + \frac{1}{2}P_R d_1\sin(\alpha(k) - \alpha_d(k)) \\ &- \frac{1}{2}P_I d_1\cos(\alpha(k) - \alpha_d(k)). \end{aligned} \tag{12}$$

Assuming that the phase error $\alpha - \alpha_d$ is small, the result is obtained.
Comments: Equation (9) can be viewed as an alternative description of the plant. Note that

$$\alpha(k) - \alpha_d(k) = \alpha(k-1) - \alpha_d(k-1) + \theta_2(k-1) - \omega_1 \tag{13}$$

so that, with the change of variables, the plant is approximately a linear time-invariant plant with two inputs $\theta_1(k)$ and $\theta_2(k)$ and two outputs $y_1(k)$ and $y_2(k)$. The transfer function matrix consists of a gain matrix G and a discrete-time integrator in the second channel. The parameters d_1 and ω_1 act as constant disturbances applied to the inputs. The parameter d_1 also appears as a gain in the second channel. In contrast to (9), equation (12) constitutes a *nonlinear* approximation of the plant. This nonlinear approximation is useful to understand the transient properties of the algorithm, but is not used in this paper.

The elimination of the high-frequency components in the signals $y_1(k)$ and $y_2(k)$ can be achieved by low-pass filtering. In the algorithm discussed in this paper, the signals are applied to a compensator which is itself low-pass. Thus, no filter was used for the simulations. However, it may be added if needed.

2.3 Compensator Design

Define two variables w_1, w_2 through

$$\left[\begin{array}{c} w_1 \\ w_2 \end{array}\right] = G^{-1}\left[\begin{array}{c} y_1 \\ y_2 \end{array}\right], \tag{14}$$

so that the dynamics of the system become decoupled. Specifically, in the z-domain

$$\begin{aligned} w_1 &= \theta_1 - {\theta_1}^* \\ w_2 &= \frac{d_1}{z-1}\ [\theta_2 - {\theta_2}^*]. \end{aligned} \tag{15}$$

The control law is chosen to be the cascade of the transformation (14) and compensators

$$\theta_1 = \frac{C_1(z)}{z-1}[w_1], \qquad \theta_2 = \frac{C_2(z)}{z-1}[w_2]. \tag{16}$$

The poles at $z = 1$ are included in the compensators to guarantee the rejection of the constant disturbances composed of d_1 and ω_1 (the compensators are selected as conventional controllers with integral compensation). The compensator transfer functions $C_1(z)$ and $C_2(z)$ are designed to guarantee the stability of the closed-loop systems and satisfactory transient responses. For example, one can choose

$$C_1(z) = -g_1, \qquad C_2(z) = -g_2\frac{z - z_a}{z - z_b}, \tag{17}$$

where g_1, g_2, z_a, and z_b are design parameters. The overall adaptive algorithm is then given by (5), (6), (8), (14), (16), and (17).

Two simple tuning methods are proposed for the parameters of the control law. The first is the equivalent of the continuous-time control law of [7]. Let z_d be some desirable location to place the closed-loop poles. For the first channel, the objective leads to a parameter $g_1 = 1 - z_d$. For the second channel, the parameters are

$$g_2 = \frac{3(1 - z_d)^2}{d_1}, \qquad z_a = \frac{z_d + 2}{3}, \qquad z_b = 3z_d - 2. \tag{18}$$

The second method is an alternative proposed specifically for the discrete-time version of the algorithm. It consists in placing the pole of the compensator for the second channel at the origin and neglecting the effect of the pole. The remaining closed-loop poles are both placed at z_d, giving the following parameters

$$g_2 = \frac{2(1 - z_d)}{d_1}, \qquad z_a = \frac{z_d + 1}{2}, \qquad z_b = 0. \tag{19}$$

The response of the algorithm with the second tuning method was found to be faster and with a wider convergence range, but also with a higher sensitivity to noise.

Estimates of the magnitude of the disturbance and of its frequency are required. These estimates are used as initial conditions for the variables θ_1 and θ_2, and also for the design of the compensators. The estimate of the magnitude of the disturbance d_1 is used to adjust the gain g_2 of the compensator $C_2(z)$ and the estimate of the frequency of the disturbance is used to adjust the gain matrix G^{-1}. The variables g_2 and G^{-1} may be kept constant during operation, or they may be updated as functions of the variables θ_1 and θ_2. In experiments with this algorithm [9], the frequency response of the plant was estimated automatically in an initial tuning phase, and the matrix G was updated by linear interpolation of a table look-up as a function of the estimated frequency. In the simulations of this paper, fixed values were used. Generally, the frequency of the disturbance may be estimated using an estimation algorithm (such as the one discussed in [7], or those of [12]). The magnitude may be estimated using the measured RMS output, or a fast Fourier transform. However, the region of convergence of the adaptive scheme is generally large enough that estimates based on prior information are typically sufficient.

3 Noise Analysis

3.1 Linear Approximation

The previous derivation considered the noiseless case, which resulted in nominal values for the parameters $\theta_1^* = d_1$, $\theta_2^* = \omega_1$, and nominal functions $\alpha^*(k) = \alpha_d(k)$, $u^*(k) = d_1 \cos(\alpha_d(k))$, $y^*(k) = 0$. Now, the effect of a measurement noise $n(k)$ added to the output $y(k)$ is evaluated. The idea of the derivation is to assume that the noise is sufficiently small that second-order effects can be neglected and to decompose the noise into in-phase and quadrature components.

Define $\theta_1 = \theta_1^* + \delta\theta_1$, $\theta_2 = \theta_2^* + \delta\theta_2$, $\alpha = \alpha^* + \delta\alpha$, $u = u^* + \delta u$, $y = y^* + \delta y$, $\bar{y} = y^* + \delta y + n$, where y is the actual output of the plant, and $\bar{y}$ is the measured output (the one used by the algorithm). Since $y^* = 0$, $y = \delta y$ and $\bar{y} = \delta\bar{y}$, where $\delta\bar{y} = \delta y + n$. Define components of the measured output

$$\begin{bmatrix} \bar{y}_1(k) \\ \bar{y}_2(k) \end{bmatrix} = \begin{bmatrix} \bar{y}(k)\cos(\alpha(k)) \\ -\bar{y}(k)\sin(\alpha(k)) \end{bmatrix}. \tag{20}$$

Again, these two variables are the ones that are used by the algorithm, instead of y_1, y_2, which are not available but remain as defined previously for the purpose of analysis. For the noise, we introduce the *in-phase* and *quadrature* components

$$\begin{bmatrix} n_1(k) \\ n_2(k) \end{bmatrix} = \begin{bmatrix} n(k)\cos(\alpha_d(k)) \\ -n(k)\sin(\alpha_d(k)) \end{bmatrix}. \tag{21}$$

From (20),

$$\begin{aligned} \bar{y}_1(k) &= (y(k) + n(k))\cos(\alpha_d(k) + \delta\alpha(k)), \\ \bar{y}_2(k) &= -(y(k) + n(k))\sin(\alpha_d(k) + \delta\alpha(k)). \end{aligned} \tag{22}$$

Assuming that the noise is small and neglecting second-order effects, it follows that

$$\begin{aligned} \bar{y}_1(k) &= y(k)\cos(\alpha_d(k)) + n_1(k), \\ \bar{y}_2(k) &= -y(k)\sin(\alpha_d(k)) + n_2(k). \end{aligned} \tag{23}$$

The control input is given by

$$u(k) = (d_1 + \delta\theta_1(k))\cos(\alpha_d(k) + \delta\alpha(k)), \tag{24}$$

so that, approximately,

$$\delta u(k) = \cos(\alpha_d(k))\delta\theta_1(k) - d_1 \sin(\alpha_d(k))\delta\alpha(k). \tag{25}$$

Assuming slow variations of $\delta\theta_1$ and $\delta\alpha$, the output of the plant corresponding to this input is given by

$$\begin{aligned} y(k) = {} & P_R \cos(\alpha_d(k))\delta\theta_1(k) - P_R d_1 \sin(\alpha_d(k))\delta\alpha(k) \\ & -P_I \sin(\alpha_d(k))\delta\theta_1(k) - P_I d_1 \cos(\alpha_d(k))\delta\alpha(k). \end{aligned} \tag{26}$$

Multiplication of this signal by $\cos(\alpha_d)$ and $\sin(\alpha_d)$, and application to (23), yields signals whose first-order/low-frequency components are given by

$$\begin{bmatrix} \bar{y}_1(k) \\ \bar{y}_2(k) \end{bmatrix} = G \begin{bmatrix} \delta\theta_1(k) \\ d_1 \delta\alpha(k) \end{bmatrix} + \begin{bmatrix} n_1(k) \\ n_2(k) \end{bmatrix}. \tag{27}$$

The result shows that, to first-order, the effect of the measurement noise is equivalent to the addition of two noise variables in the transformed system. Again, analysis can again be performed in a linear time-invariant framework.

3.2 Computation of Variances

A transfer function matrix relates the noise sources to the parameter deviations, with

$$\begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \\ \delta\alpha \end{bmatrix} = \begin{bmatrix} \frac{C_1(z)}{z-1-C_1(z)} & 0 \\ 0 & \frac{(z-1)C_2(z)}{(z-1)^2-d_1C_2(z)} \\ 0 & \frac{C_2(z)}{(z-1)^2-d_1C_2(z)} \end{bmatrix} G^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}. \tag{28}$$

To apply the results and estimate the performance of the algorithm in the presence of noise, the power spectra of the noise components n_1 and n_2 must be known. If n is a white noise with variance σ^2, it is common to assume, in the analysis of phase-locked loops, that n_1 and n_2 are uncorrelated white noises with variances equal to $\frac{1}{2}\sigma^2$ [10]. We will make the same assumption. The power spectra of $\delta\theta_1$, $\delta\theta_2$, and $\delta\alpha$ can then be deduced using the transfer function (28), and the variances of these variables can be computed by integration of the power spectra.

Alternatively, a Lyapunov equation may be solved, using the following state-space description

$$x(k+1) = Ax(k) + BG^{-1}v(k), \tag{29}$$

where

$$A = \begin{bmatrix} 1-g_1 & 0 & 0 & 0 \\ 0 & 1+z_b & 1 & -g_2d_1 \\ 0 & -z_b & 0 & g_2z_ad_1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -g_1 & 0 \\ 0 & -g_2 \\ 0 & g_2z_a \\ 0 & 0 \end{bmatrix},$$

$$x(k) = \begin{bmatrix} \delta\theta_1(k) \\ \delta\theta_2(k) \\ \delta\theta_3(k) \\ \delta\alpha(k) \end{bmatrix}, \quad v(k) = \begin{bmatrix} n_1(k) \\ n_2(k) \end{bmatrix}. \tag{30}$$

Note that these equations, as well as previous equations, assume that the G matrix used by the compensator is the true matrix corresponding ot the plant. However, the analysis can be extended to account for a difference in the true and estimated matrices. Defining $V = E(vv^T) = \frac{1}{2}\sigma^2 I$, and assuming that n_1 and n_2 are white noise sources, the value of the steady-state covariance matrix $P = E(xx^T)$ is given by the solution of the discrete Lyapunov equation ([11], p. 471)

$$AXA^T + BG^{-1}V(G^{-1})^TB^T = X. \tag{31}$$

which is easily obtained using modern mathematical software.

Regarding the output signals y and $\bar{y}$, (26) shows that y is a sinusoidal function with the same frequency as the disturbance, but with a peak and phase that are random processes. The variance of the magnitude of the sinusoid is equal to $({P_R}^2+{P_I}^2)(E(\delta{\theta_1}^2)+{d_1}^2E(\delta\alpha^2))$. Although the variance of the plant output is a function of time, its average over the period of the disturbance may be defined. This average variance may be estimated experimentally or in simulations, and is a significant performance measure for the algorithm. Using the previous results, one finds that the average variances of the true and measured plant outputs are given by

$$\begin{aligned} E_{avg}(y^2) &= \frac{({P_R}^2+{P_I}^2)}{2}(E(\delta{\theta_1}^2)+{d_1}^2E(\delta\alpha^2)) \\ E_{avg}(\bar{y}^2) &= E_{avg}(y^2)+\sigma^2. \end{aligned} \tag{32}$$

The subscript avg refers to the averaging of the variances over the period of $\alpha_d(k)$.

4 Simulation Results

We present simulation results obtained for a plant which is a pure delay $P(z) = z^{-10}$. The disturbance has magnitude $d_1 = 1$, a period equal to 100 time samples,. The initial phase was set at $\delta_1 = 0$, but could be set at arbitrary values. The estimates used by the algorithm are 0.8 for the magnitude and 120 steps for the period. The desired closed-loop pole was selected to be $z_d = 0.99$, leading to $g_1 = 0.01$, $g_2 = 0.025$, $z_a = 0.995$, and $z_b = 0$ (for the second tuning method). These parameters were left fixed throughout the simulations.

Fig. 2 shows the measured output of the plant ($\bar{y}$). The plot on the left corresponds to a simulation for a low noise condition: $\sigma = 0.01$, with the uncompensated plant output being a sinusoid with magnitude equal to 1. The plot on the right corresponds to a high noise condition, with $\sigma = 0.5$ (or a standard deviation equal to 50% of the magnitude of the uncompensated plant output, instead of 1%). Figs. 3 and 4 show the responses of the magnitude and frequency estimates (θ_1 and θ_2), showing convergence towards the expected values (the nominal magnitude is 1 and the nominal frequency is $2\pi/100 = 0.0628$). In the high noise plots, the convergence of the parameters towards their nominal values is barely visible in the fluctuations induced by the noise. However, the output is significantly smaller than what it would be under uncompensated conditions, as shown in Fig. 5.

The predicted variances were computed by solving the discrete Lyapunov equation. The results were compared to sample variances obtained from the simulations by averaging deviations between $k = 1000$ and $k = 11000$ (in other words, a longer time period was used than for the plots, and the initial transient was left out of the computations). The results are summarized in Table 1. The standard deviations predicted by the solution of the Lyapunov equation are given under the heading of "analysis", while the results obtained through averaging of the simulations results are shown as "simulation." The numbers show a good match between the predictions of the analysis and the values observed in simulations. One also finds that the variation of the measured output is primarily made of the noise itself, rather than a variation induced by the fluctuations of the adaptive parameters.

Although the analysis of the paper and the simulations emphasized the steady-state performance of the algorithm, significant changes in the disturbance parameters, as well as abrupt changes, may be accommodated by the algorithm. Fig. 6 shows the results of a simulation with low noise over 3000 samples, with the magnitude and the frequency of the disturbance increased by 50% at $k = 1000$

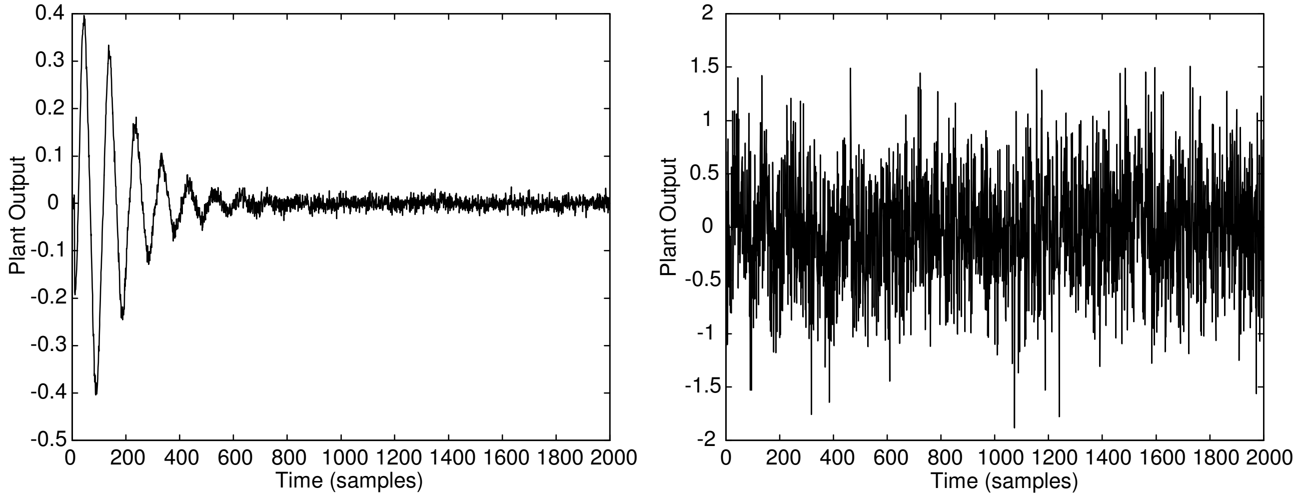

Figure 2: Measured Plant Output - Low Noise (Left), High Noise (Right)

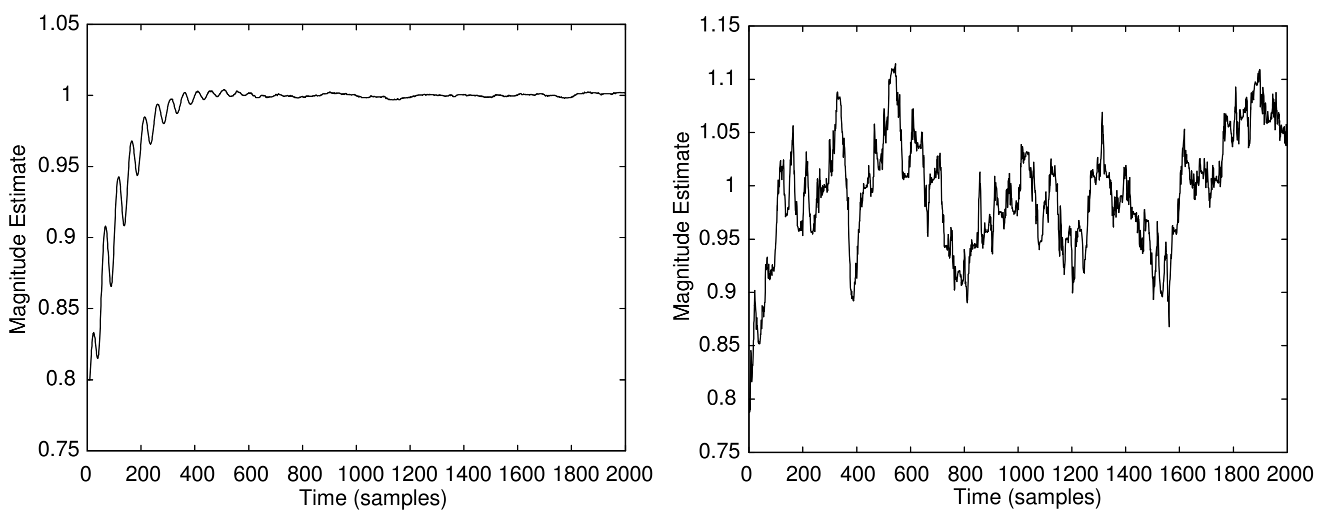

Figure 3: Magnitude Estimate - Low Noise (Left), High Noise (Right)

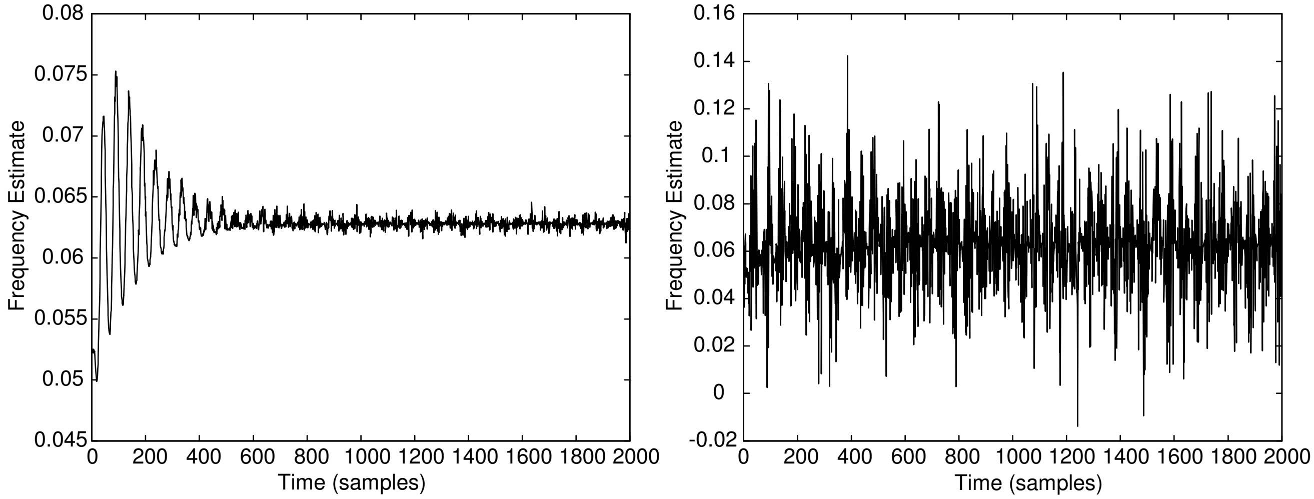

Figure 4: Frequency Estimate - Low Noise (Left), High Noise (Right)

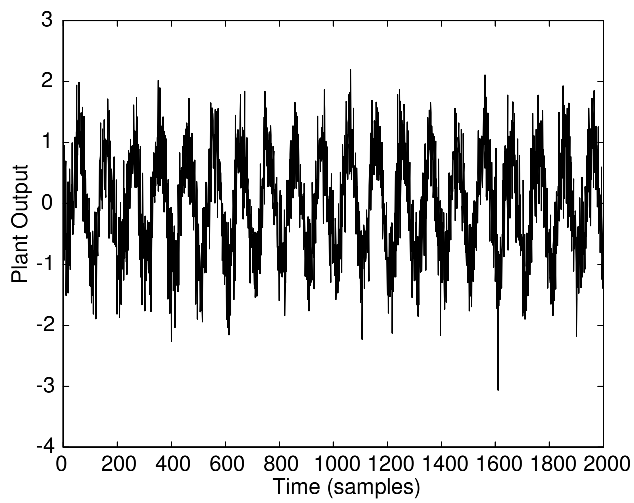

Figure 5: Measured Plant Output without Compensation and with High Noise

	y	$\bar{y}$	$\delta\theta_1$	$\delta\theta_2$
$\sigma = 0.01$				
Analysis	0.0014	0.0101	0.0010	3.5610^{-4}
Simulation	0.0016	0.0103	0.0011	3.6510^{-4}
$\sigma = 0.5$				
Analysis	0.0718	0.5051	0.0501	0.0178
Simulation	0.0881	0.5110	0.0613	0.0180

Table 1: Standard Deviations of Plant Output and Parameters – Analysis *vs.* Simulation

and $k = 2000$, respectively. At $k = 1000$, the phase also jumps by 180° (the maximum possible). The figure shows the response of the magnitude estimate (left) and of the frequency estimate (right). After some transient responses, both estimates converge to their desired values.

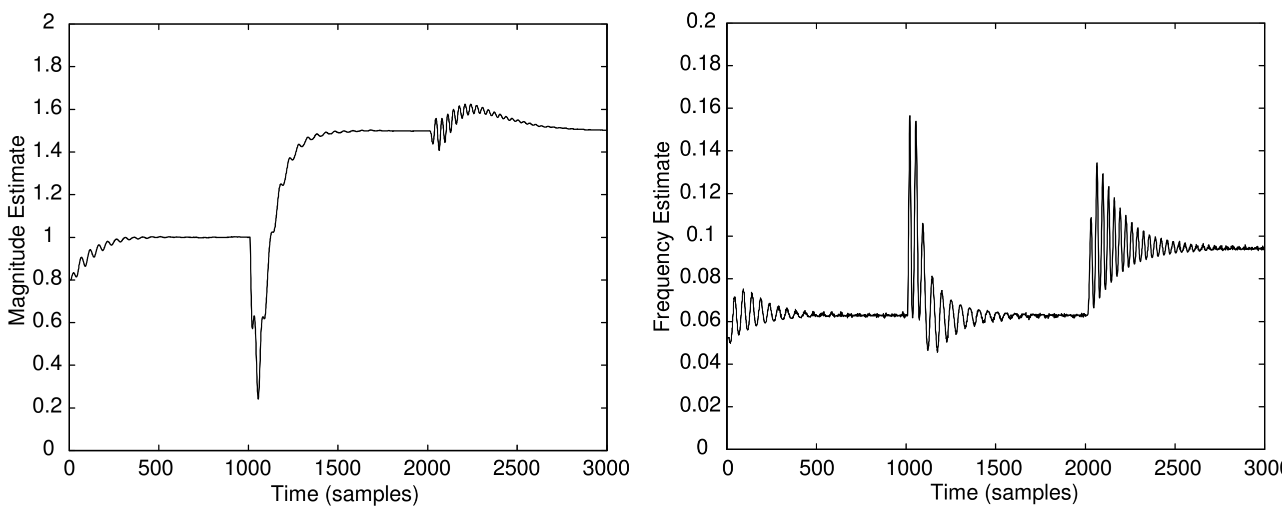

Figure 6: Adaptive Parameter Responses to Sudden Changes in the Disturbance Parameters

5 Conclusions

In this paper, we discussed a method for the rejection of sinusoidal disturbances with unknown frequency. The discrete-time algorithm was obtained from an existing continuous-time algorithm, with some non-trivial adjustments. Further extensions to handle multiple harmonics are possible. The design of the control law could be conveniently carried out using linear techniques. In addition, a separate analysis predicted the loss of performance incurred in the presence of measurement noise, and the amount of fluctuation induced in the adaptive parameters. Through simulations, it was found that the algorithm was not only effective at rejecting sinusoidal disturbances in the presence of high noise, but also that its performance could be accurately predicted. The derivations relied on a phase-locked loop technique that decomposes the noise signal into in-phase and quadrature components. Such an approach has not been widely used in control, but was found helpful for this problem. The results that were obtained do not have counterparts for other algorithms proposed in the literature for the rejection of periodic disturbances of unknown frequency.

References

[1] M. Bodson, A. Sacks & P. Khosla, "Harmonic Generation in Adaptive Feedforward Cancellation Schemes," *IEEE Trans. on Automatic Control*, vol. 39, no. 9, 1994, pp. 1939-1944.

[2] S.R. Hall & N.M. Wereley, "Performance of Higher Harmonic Control Algorithms for Helicopter Vibration Reduction," *J. Guidance, Control and Dynamics,* vol. 16, no. 4, 1993, pp. 793-797.

[3] A. Sacks, M. Bodson, & P. Khosla, "Experimental Results of Adaptive Periodic Disturbance Cancellation in a High Performance Magnetic Disk Drive," *ASME Journal on Dynamics and Control,* vol. 118, 1996, pp. 416-424.

[4] K.S. Narendra and A. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1989.

[5] G. Feng and M. Palaniswamy, "A Stable Adaptive Implementation of the Internal Model Principle," *IEEE Trans. on Automatic Control*, vol. 37, no. 8, 1992, pp. 1220-1225.

[6] B. Francis & M. Vidyasagar, "Linear Multivariable Regulation with Adaptation: Tracking Signals Generated by Models with Unknown Parameters", *Proc. of the IEEE Conference on Decision and Control,* 1978.

[7] M. Bodson & S. Douglas, "Adaptive Algorithms for the Rejection of Periodic Disturbances with Unknown Frequency," *Automatica,* vol. 33, no. 12, 1997, pp. 2213-2221.

[8] T.J. Manayathara, T.-C. Tsao, J. Bentsman, & D. Ross, "Rejection of Unknown Periodic Load Disturbances in Continuous Steel Casting Process Using Learning Repetitive Control Approach," *IEEE Trans. on Control Systems Technology,* vol. 4, no. 3, 1996, pp. 259-265.

[9] M. Bodson, J. Jensen, & S. Douglas, "Active Noise Control for Periodic Disturbances," *Proc. of the American Control Conference,* Philadelphia, PA, 1998, pp. 2616-2620.

[10] D.H. Wolaver, *Phase-Locked Loop Circuit Design,* Prentice-Hall, Englewood Cliffs, New Jersey, 1991.

[11] H. Kwakernaak & R. Sivan, *Linear Optimal Control Systems*, Wiley, New York, 1972

[12] P. Stoica & R. Moses, *Introduction to Spectral Analysis*, Prentice-Hall, Upper Saddle River, NJ, 1997.