Spontaneous Speed Reversals in Stepper Motors

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Abstract

Experimental data shows that permanent magnet stepper motors can spontaneously reverse their direction of rotation when controlled in full step, open-loop mode. The paper shows that the reversal of speed can be explained, and that the reverse speed is equal to three times the forward speed. A linear approximation of the motor dynamics predicts that reverse running will arise when the undershoot of the single-step response exceeds 50% (or when the damping factor is less than 0.11). The observations of the paper suggest a revision of the conventional explanation that resonance problems occur because of high positive velocity at the stepping time. Instead, the data and the analysis of the paper suggest that resonance problems, including stalling and speed reversal, occur due to the growth of the undershoot in the response to repeated steps.
Keywords: stepper motor, stepping motor, synchronous motor, resonance, stall, reverse motion, nonlinear dynamics.

1. Introduction

The paper describes a phenomenon in which the speed of a permanent magnet (PM) stepper motor reverses unexpectedly. Fig. 1 shows data collected on a stepper motor at

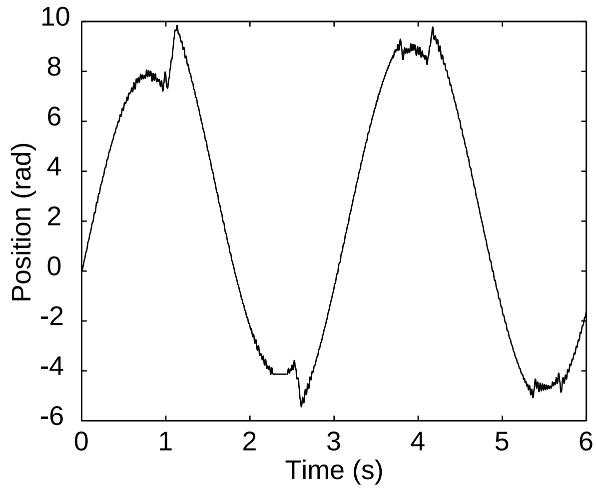

Figure 1: Speed reversal of a stepper motor

the University of Utah. The stepper motor is a two-phase permanent magnet motor with 7.5° step size (*i.e.,* 12 pole pairs). The motor was commanded in open-loop, full stepping mode, with a voltage-source-amplifier and a sinusoidal reference profile. The figure shows several instances where the speed changes direction with respect to the command. A similar phenomenon was observed at L-3 Communications with a hybrid stepper motor (1.8° step size) under current command.

Stepper motors controlled in open-loop are known to be susceptible to resonances at step rates equal to the frequency of resonance and to its subharmonics. A speed reversal phenomenon was reported in [5] for a variable-reluctance stepper motor. Fig. 1 indicates that this phenomenon can also be observed with PM stepper motors, although such motors are less susceptible to resonance problems. In this paper, we show how the phenomenon can be explained, and how its onset can predicted. The analysis suggests a revision to the common interpretation of low-speed resonance in stepper motors.

2. Explanation of speed reversal

The possibility of speed reversal is counter-intuitive, since the stator currents create a

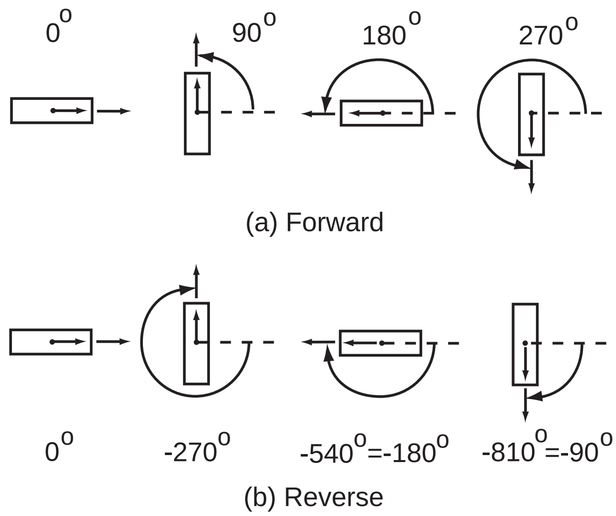

Figure 2: Principle of speed reversal at three times the forward speed

rotating magnetic field in a given direction. The existence of the phenomenon is tied to the stepping nature of the rotating field. Consider Fig. 2, where a permanent magnet motor with one pole pair is shown. Arrows show the directions of the rotor and of the stator magnetic fields. Assuming a two-phase motor, part (a) of the figure shows 4 steps of the forward motion of the rotor under full stepping commands. Part (b) shows how a reverse motion with the same equilibrium positions is theoretically possible. Note that the resulting reverse speed is 3 times the forward speed. The 3x reversal speed is consistent with data obtained at the University of Utah and with observations made at L-3 Communications on a separate system.

The data reported in [5] (also reproduced in [4]) for a three-stack variable-reluctance motor indicates a 2x speed reversal. Note that our explanation of the speed reversal, appropriately adjusted, predicts a 2x reversal speed for a 3-phase motor. The same explanation also predicts the possibility of 5x, 9x, ..., forward speeds, and 7x, 11x, ... reverse speeds for a two-phase PM motor. The phenomenon is analogous to aliasing in sampled-data systems. Sustained operation at speeds other than the nominal forward

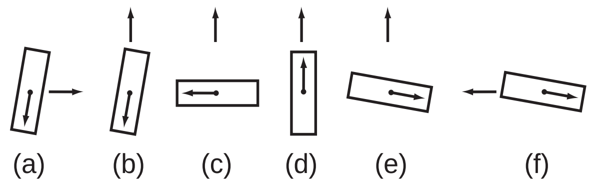

Figure 3: Intermediate positions of reverse motion

speed, however, requires that the dynamics of the system permit transition from one equilibrium to the next along the pattern shown in Fig. 2. It is not immediately obvious how the rotor can move from the $0°$ to the $-270°$ position, since the torque acts in the counterclockwise direction.

Our investigation of the phenomenon has led us to conclude that, for reverse motion to be possible, the rotor must lag behind the stable equilibrium position by approximately $90°$ at the end of a step in the sequence. Fig. 3 explains how reverse motion can then be sustained. Stage (a) shows the position of the rotor at the end of a step, with the rotor lagging by slightly more than $90°$ behind the direction of the stator magnetic field. Torque is positive, but when the next step starts at stage (b), the angle becomes greater than $180°$, yielding a small negative torque. The motor will move in the negative direction. Stage (c) shows an intermediate position along that path, where the negative torque has maximum magnitude. Note that it may take some time to reach position (c), since the torque is initially small. At (d), the torque is again zero, but the rotor has acquired a sufficient velocity that it overshoots the equilibrium. The torque becomes positive and the motor slows down while reaching position (e). At that time, the next step in the sequence is reached and the cycle repeats, with (f) equivalent to (b). In this manner, reverse motion with a negative average torque is possible. Note that the reverse motion is also possible if the initial angle is less than $90°$, provided that the motor has a small negative velocity, since the torque is small after switching.

It remains to be shown that the sequence of Fig. 3 can be initiated, *i.e.*, that a 90° lag can be obtained. This is the subject of the next two sections.

3. Stepper motor model and characteristics

Consider a two-phase permanent magnet (PM) stepper motor under current command. Denote the currents flowing in the stator windings i_A and i_B (in A). Let θ be the angle of the rotor (in rad), so that $\theta = 0$ when the PM flux in winding A is maximized. Assuming linear magnetics and a sinusoidal PM flux in the windings, the torque τ_e (in N.m) produced by the motor is given by

$$\tau_e = -K i_A \ \sin(n_p \theta) + K i_B \ \cos(n_p \theta) \tag{1}$$

where K is the torque constant (in N.m/A) and n_p is the number of pole pairs (or rotor teeths) of the motor. In practice, the torque is not exactly sinusoidal, and reluctance effects (such as the detent torque) affect the overall torque. However, the approximation is adequate for our analysis.

The torque can also be written as

$$\tau_e = KI \ \sin\left(\theta_e - n_p \theta\right) \tag{2}$$

where

$$I = \sqrt{i_A^2 + i_B^2}, \quad \theta_e = \measuredangle(i_A, i_B) \tag{3}$$

I is the magnitude of the 2-dimensional current vector with components i_A and i_B, and θ_e is the angle of the vector with respect to the x-axis. Open-loop control of position consists in applying currents such that the angle θ_e is varied by discrete increments. The full step two-phases-on method corresponds to the sequence

θ_e	i_A	i_B
45°	I_0	I_0
135°	$-I_0$	I_0
225°	$-I_0$	$-I_0$
315°	I_0	$-I_0$

(4)

The sequence produces equilibrium positions at 45°, 135°, ..., separated by 90 electrical degrees.

The dynamic response of a motor can be described by

$$J\frac{d^2\theta}{dt^2} = -B\frac{d\theta}{dt} + KI\ \sin\left(\theta_e - n_p\theta\right) \tag{5}$$

where J is the inertia of the motor, and B is a constant associated with viscous friction. Static friction and load effects (except for the load inertia that may be added to the inertia of the motor) have been neglected. (5) is a nonlinear differential equation. However, if the rotor stays close to the equilibrium position defined by the current vector (*i.e.*, $|\theta_e - n_p\theta|$ is small), an approximate linear differential equation can be obtained

$$J\frac{d^2\theta}{dt^2} = -B\frac{d\theta}{dt} + KI\left(\theta_e - n_p\theta\right) \tag{6}$$

The poles of the system are given by the roots of

$$s^2 + \frac{B}{J}s + \frac{KIn_p}{J} = 0 \tag{7}$$

Because viscous friction is small in stepper motors, the roots are generally poorly damped. If θ_e suddenly changes, the response of the motor is oscillatory. The resonant frequency (Hz) of the system can then be estimated from

$$f_r = \frac{1}{2\pi}\sqrt{\frac{KIn_p}{J}} \tag{8}$$

Fig. 4 shows the two-phases-on step response of a stepper motor at the University of Utah. The response was obtained under voltage command, which generally provides more damping than current command. Yet, significant overshoot and multiple oscillations are observed.

4. Resonance and initiation of reverse motion

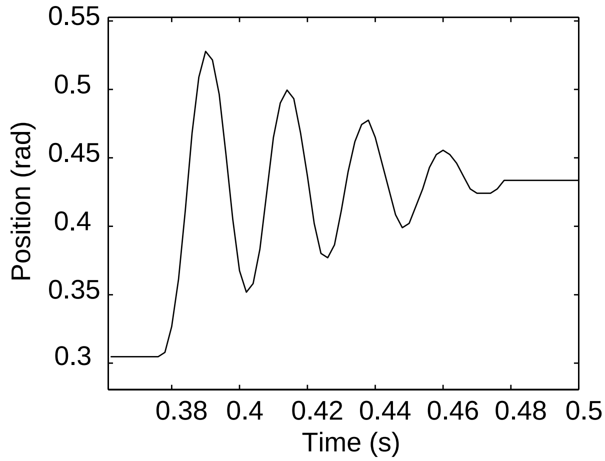

Figure 4: Motor step response

Consider a linear time-invariant system with poles at $s = -a \pm jb$ and with unity DC gain

$$H(s) = \frac{Y(s)}{X(s)} = \frac{a^2 + b^2}{s^2 + 2as + a^2 + b^2} \tag{9}$$

The response of the system to a unit step input ($x(t) = 1$) is given by

$$y(t) = 1 - e^{-at}\cos(bt) - e^{-at}\sin(bt) \tag{10}$$

For a/b small, the response is oscillatory. Peaking occurs for

$$\begin{aligned} t_1 &= \frac{\pi}{b}, \qquad y(t_1) = 1 + e^{-\frac{a\pi}{b}} \\ t_2 &= \frac{2\pi}{b}, \qquad y(t_2) = 1 - e^{-\frac{2a\pi}{b}} \end{aligned} \tag{11}$$

and so on.

The principle of resonance is shown in Fig. 5 for $a = 0.1$, $b = 1$. If a second step is applied at time $t_2 = 2\pi/b$, the response of the system for $t > t_2$ (shown as a solid curve) is the sum of the response due to the first step and the response due to the second step (both shown as dashed curves). Note that, due to the timing of the second step, the

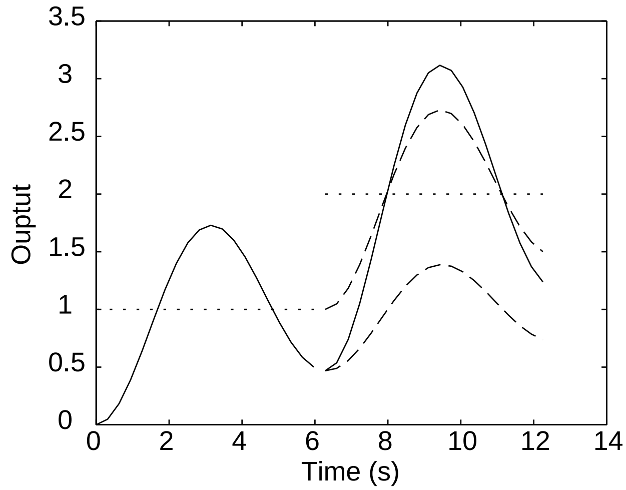

Figure 5: Response of second-order system to double step at the frequency of resonance

overshoot is considerably magnified after the second step. The positive peak becomes

$$y(\frac{3\pi}{b}) = 2 + e^{-\frac{a\pi}{b}} + e^{-\frac{3a\pi}{b}} \tag{12}$$

Asymptotically, the *percent overshoot (PO)* is given by

$$\frac{PO(\%)}{100} = e^{-\frac{a\pi}{b}} + e^{-\frac{3a\pi}{b}} + \ldots = \frac{e^{-\frac{a\pi}{b}}}{1 - e^{-\frac{2a\pi}{b}}} \tag{13}$$

Fig. 5 shows that the *undershoot* is also magnified, and becomes asymptotically

$$\frac{PU(\%)}{100} = e^{-\frac{2a\pi}{b}} + e^{-\frac{4a\pi}{b}} + \ldots = \frac{e^{-\frac{2a\pi}{b}}}{1 - e^{-\frac{2a\pi}{b}}} \tag{14}$$

Note that the asymptotic undershoot reaches 100% if the first undershoot is 50%, that is, if

$$e^{-\frac{2a\pi}{b}} = 0.5, \quad \text{or } \frac{a}{b} = 0.11 \tag{15}$$

The condition is numerically similar in terms of the damping factor

$$\zeta = \frac{a/b}{\sqrt{1 + (a/b)^2}} = 0.11 \tag{16}$$

In the context of full stepping of a stepper motor, a 100% undershoot means that the motor lags the equilibrium position by one full step. If the undershoot occurs when the next step is applied, the rotor will lag the *next* equilibrium position by *two full steps* (or 180 electrical degrees). Therefore, the condition for a 100% undershoot defines a limit above which loss of synchronism will occur. Reverse motion, as shown in Fig. 3 can begin.

The nonlinear differential equation (5) predicts the growth of undershoot and the torque reversal. Fig. 6 shows the result of a simulation with multiple step inputs and with a step rate (in steps/s) equal to the resonant frequency (in Hz). One can clearly observe the build-up of the undershoot until the $-180°$ position is reached and reverse motion starts. The progression is not exactly as predicted by the linear theory, because the oscillations take the system well into the nonlinear region ($90°$ in the sin function). In practice, reverse motion may also be triggered, or delayed, by perturbations such as load or gear train oscillations. However, the linear approximation is useful to predict the likelihood of reverse motion.

Once reverse motion is started, it may continue and show up as a backward-running motor. Often, however, the motor will switch between periods of forward and reverse motions. The resulting behavior will appear as stalling. Nevertheless, stalling and steady reverse motion are two resonance problems having the same origin.

Our explanation of resonance in stepper motors is different from the common interpretation. Indeed, Fig. 7 shows a plot representative of diagrams found in [1], [3], [5]. The interpretation is that a step rate equal to the resonant frequency yields switching at a time where the rotor has high forward velocity. As the overshoot is amplified, a situation is reached where the torque changes sign and the rotor escapes in the forward direction. This interpretation does not match the data that we have collected or, for that matter, the data found in [5]. In fact, escape in the forward direction is unlikely to occur. In Fig. 7, the position at which the torque changes sign at the time of step command 3 is step

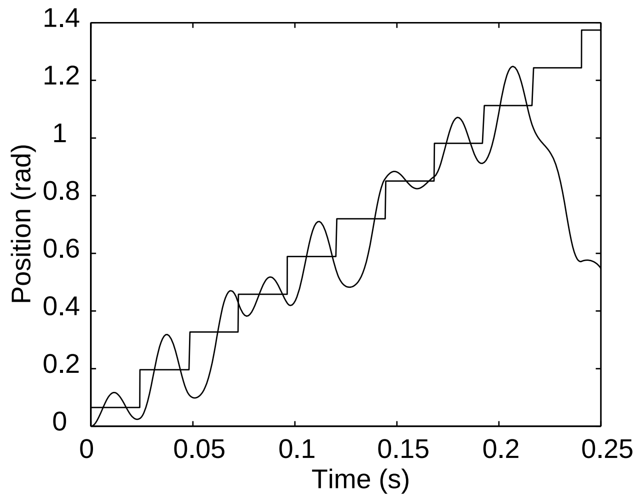

Figure 6: Simulation of stepper motor response to multiple steps

position 5. Escape in the forward direction requires an overshoot of 200%. Condition (13) indicates such an overshoot is possible, although requiring a ratio $a/b = 0.079$ that is smaller than the condition for a 100% undershoot. Note that, for poorly damped systems, the first overshoot is close in magnitude to the first undershoot. Because escape in the reverse direction occurs for a 100% undershoot, instead of the 200% overshoot required in the forward direction, escape in the reverse direction will generally occur *before* it may occur in the forward direction.

5. Experimental results

Data was collected on the motor whose step response was shown on Fig. 4 (Airpax motor #C82711-M1, 17VDC dual coil, 7.5° step). Note that the first undershoot is greater than 50%, suggesting a high risk of speed reversal. The frequency of resonance was estimated to be $f_r = 41.7$Hz. For 12 pole pairs, a step rate equal to the frequency of resonance corresponds to a speed of rotation equal to

$$\omega_{RPM} = \frac{60 f_r}{4np} = 52\text{rpm} \tag{17}$$

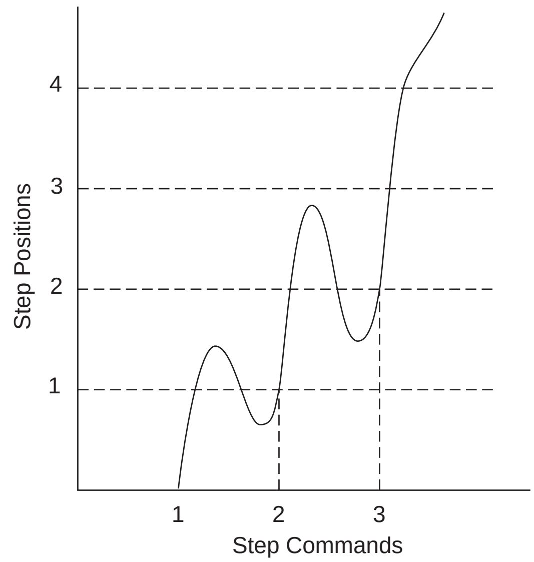

Figure 7: Typical interpretation of resonance in stepper motors

In experiments, resonances were found to occur in the range of 43 to 55rpm, approximately. The responses in the middle of the range were characterized by stalling. Fig. 8 shows a typical example at 49rpm. One can see that stalling actually results from frequent switching between forward speed and reverse speed at 3 times the forward speed.

Fig. 9 shows the response at 51rpm, which shows longer periods of forward and reverse motions. At the end of the plot, there is even a period where alternations of forward (1x) and reverse (3x) motions yield a reverse motion with an average speed close to the forward speed. As in [5], speed reversals were observed in the frequency bands bordering normal behavior, while stalling was observed in the middle of the resonance band. Also as in [5], reverse motion sometimes persisted for significant periods of time. In experiments at L-3 Communications, we have observed sustained reverse motion at 3x the forward speed for several revolutions of the motor.

Fig. 10 shows speed reversals at 25rpm, which corresponds to the first subharmonic of the resonant frequency. The same theory shows that, for sufficiently low damping, build-

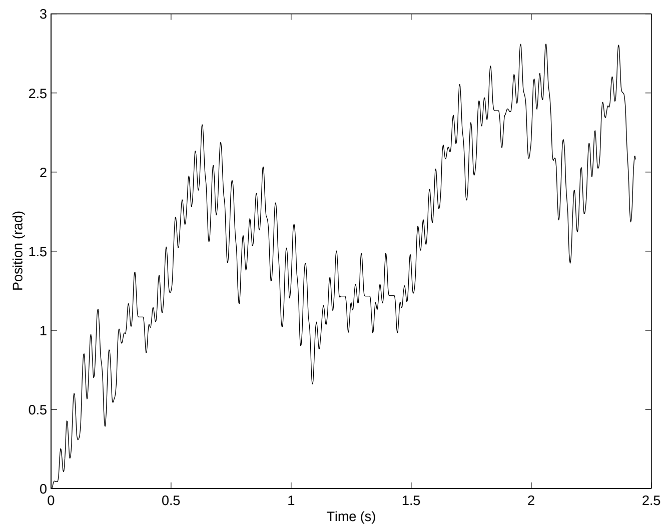

Figure 8: Stepper motor response at 49rpm

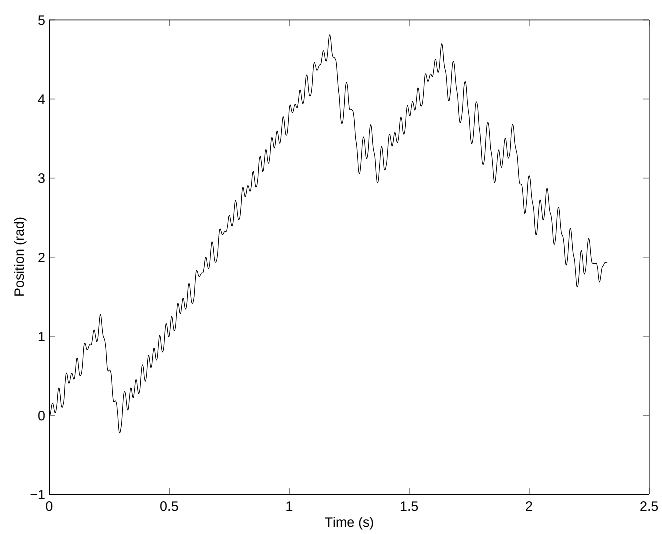

Figure 9: Stepper motor response at 51rpm

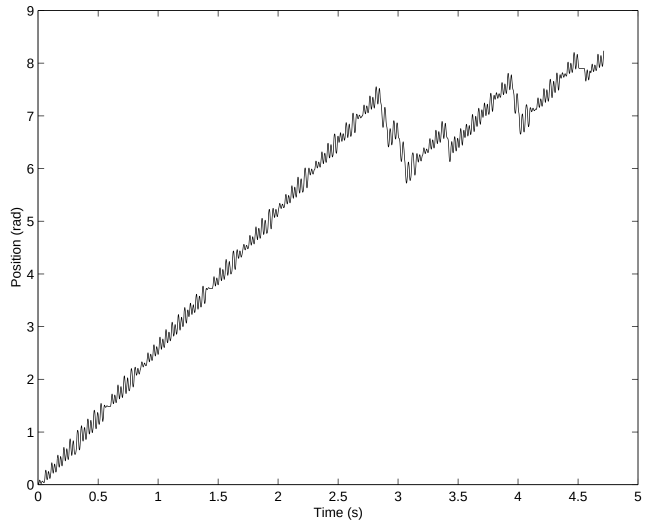

Figure 10: Stepper motor response at 25rpm

up of the second overshoot can result in loss of synchronism. The (approximate) condition is that the second overshoot must exceed 50%. Away from the resonant frequency and its first subharmonic, the motor was found to track the reference commands well. Speeds up to 250rpm could be reached in open-loop mode.

A close look at the responses of the motor shows that reverse motion follows the stages of Fig. 3. Fig. 11 shows a detail of Fig. 1, with the smooth curve showing the response of the motor. The stable equilibrium positions corresponding to the commands are shown by the stepwise curves (separated by $2\pi/n_p = \pi/6$). The figure confirms that the reverse motion occurs at three times the forward speed, and that the motor briefly settles at the *unstable* equilibrium positions at the beginning of a new step. Interestingly, the onset of this reverse motion can be predicted by a relatively simple differential equation, although it strongly depends on the nonlinear nature of the differential equation.

6. Avoiding resonances and speed reversals

Methods for avoiding low-speed resonances are well-known and include ([1], [2], [4]):

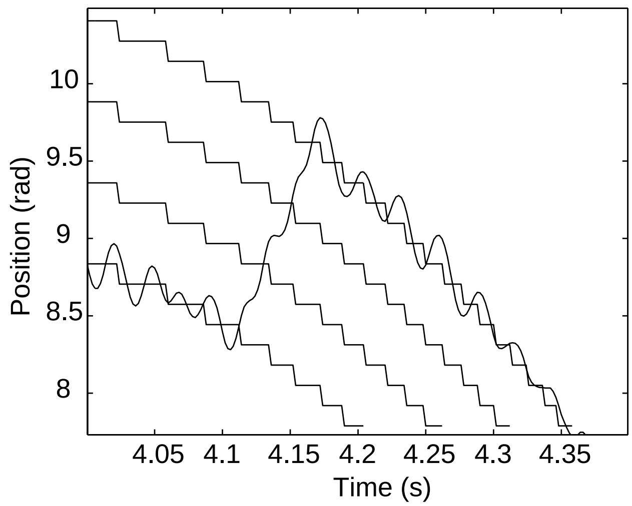

Figure 11: Detail of reversal during multiple steps

- increasing mechanical damping,
- increasing damping through electrical means,
- microstepping,
- closed-loop control.

Ofren, microstepping will be the easiest approach. Even a change from two-phases-on to one-phase-on can eliminate the problem. For the motor used in the experiments, the one-phase-on mode of stepping (with equilibrium positions at 0°, 90°, ..., instead of 45°, 135°, ...) gives the step response of Fig. 12, which is considerably more damped. The undershoot is less than 50% and resonance problems have not been observed in that mode. The two-phases-on is often preferred because of its higher torque and because of the simplicity of the power electronic circuit. However, lower damping results not only from the higher torque, but also from the detent torque that destabilizes the equilibrium directions of the two-phases-on mode.

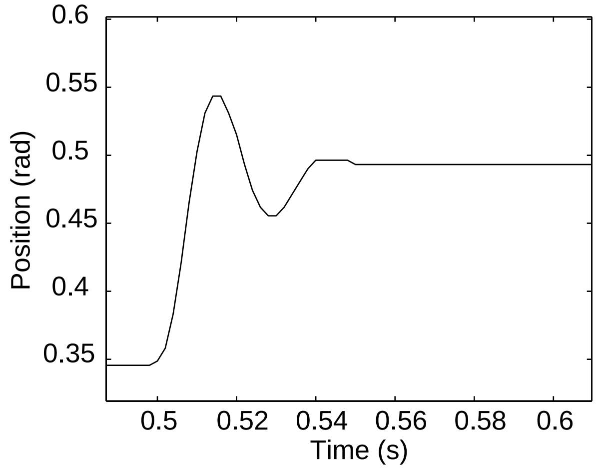

Figure 12: Step response in one-phase-on mode

7. Conclusions

The paper showed that stepper motors controlled in open-loop may reverse direction unexpectedly. Although a similar phenomenon had been observed for variable-reluctance motors, the paper went further by providing an explanation for how this behavior could be initiated and sustained. Under a linear approximation, it was found that a condition for the initiation of a speed reversal is that the undershoot in the response exceeds 50% (or a damping factor less than 0.11). The resulting reversal of the torque is the source of resonance problems including both stall and speed reversal. While the usual interpretation suggests that switching at a time of high forward velocity is the underlying cause of the reversal, the analysis and data of this paper shows that the problem originates instead in the growing undershoot.

8. References

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