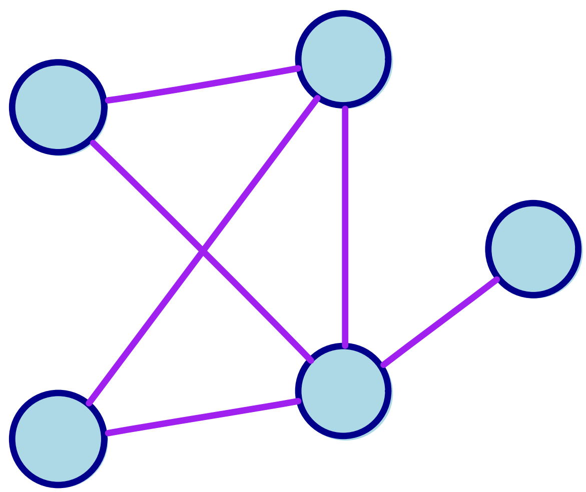
Graphs

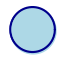
A ***graph*** is

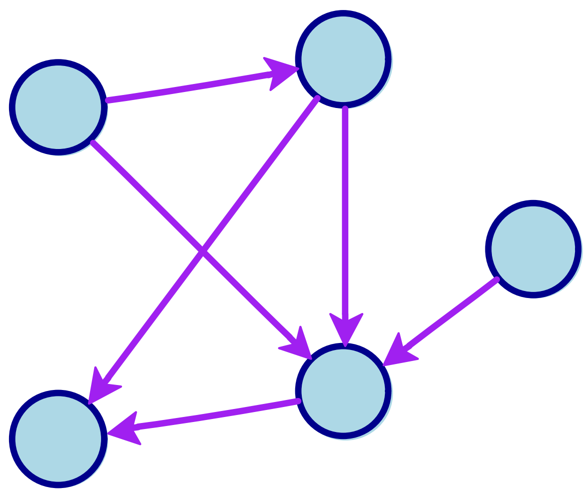
- a set of ***nodes***
- a set of ***edges***
  each connecting two nodes



Graphs

A ***directed graph*** is

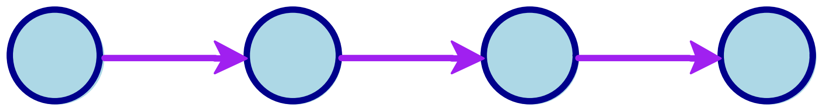
- a set of ***nodes*** 
- a set of ***edges*** →
  each connecting one node to another node



We'll just use "graph" to mean "directed graph"

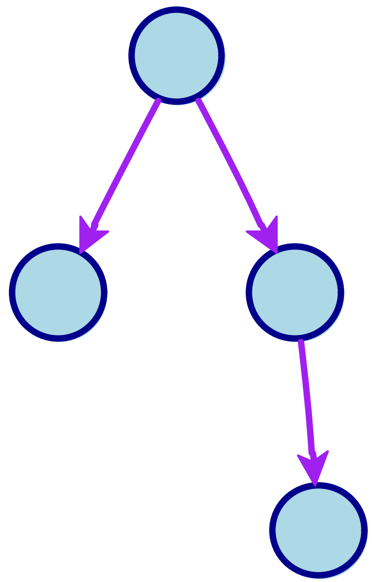
Graphs: Lists

At most one outgoing edge ⇒ ***list***



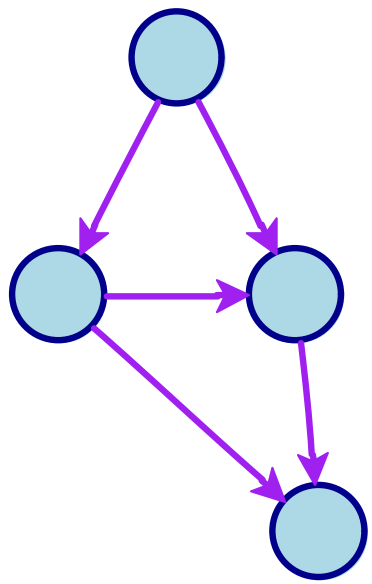
Graphs: Trees

Reach each node in only one way ⇒ ***tree***



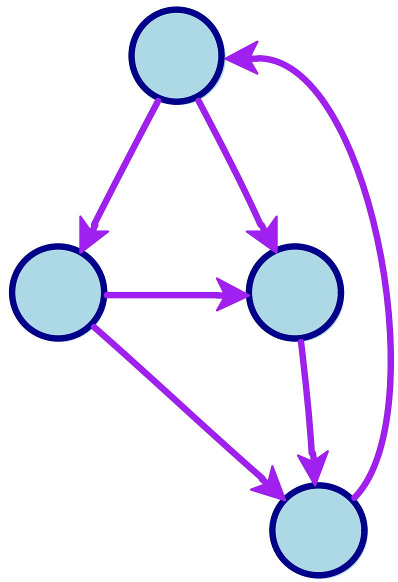
Graphs: DAG

Can't get to a node from itself ⇒
directed acyclic graph (DAG)



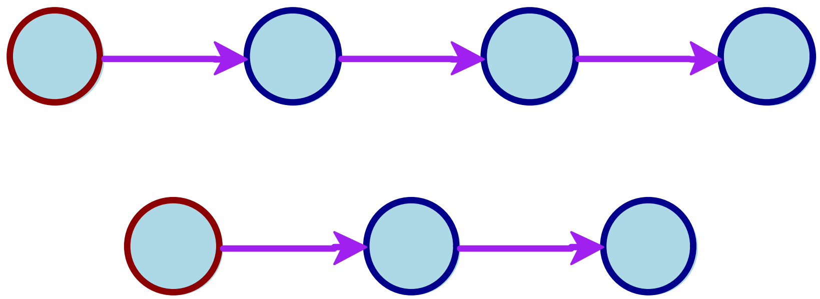
Graphs: Cycles

Can get to a node from itself ⇒ ***graph***



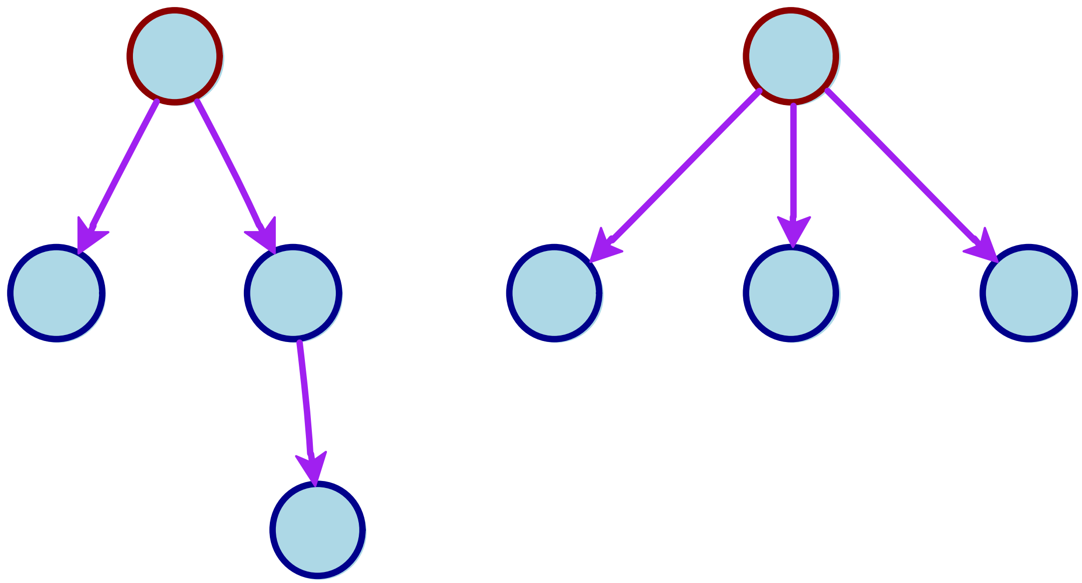
Roots

Some nodes might be considered ***roots*** — often nodes that reach all others



Roots

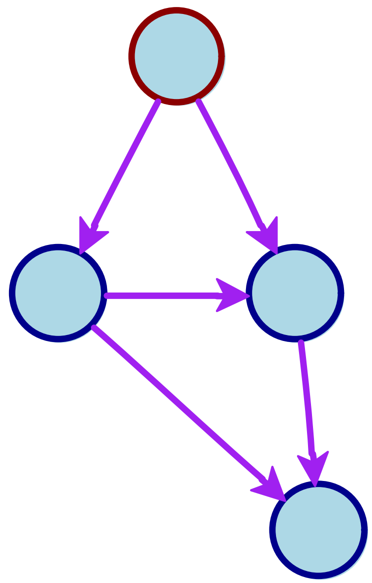
Some nodes might be considered ***roots*** — often nodes that reach all others



A graph containingly only trees is a ***forest***

Roots

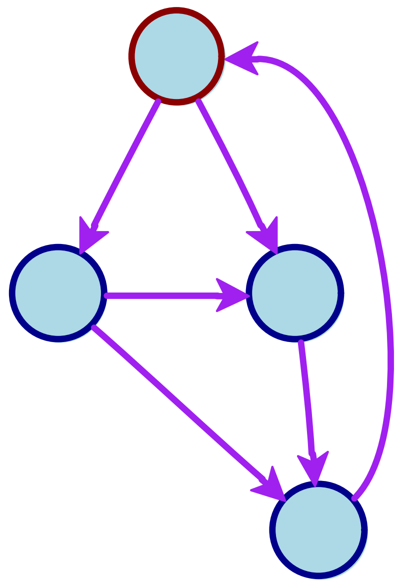
Some nodes might be considered ***roots*** — often nodes that reach all others



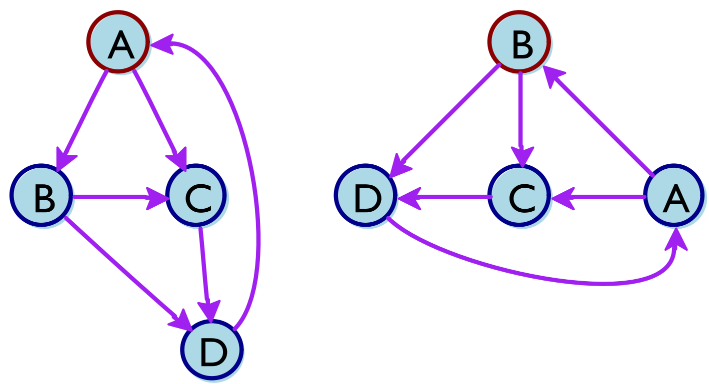
Can reach all nodes from some root ⇒ a ***connected*** graph

Roots

Some nodes might be considered ***roots*** — often nodes that reach all others



Multiple candidate roots:



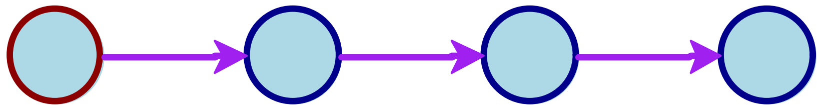
Representing Graphs

Graphs can be represented in different ways:

- Nodes as structs/objects, edges as pointers/references
- Nodes as objects, edges in a dictionary
- Nodes a integers, edges as a list of pairs of numbers

Unless you're solving abstract graph problems, typically you have an existing data definition that you might think of as a graph — probably matching the first case

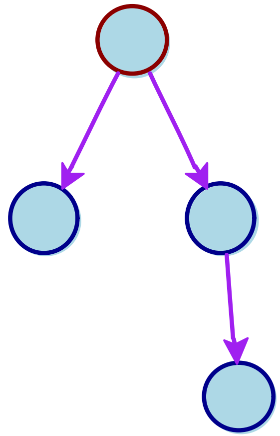
Designing Programs: Lists



```
(define (F n)
  (cond
   [(empty? n) ...]
   [else ... (F (rest n)) ...]))
```

```
for (n = root;
     n != NULL;
     n = n->next) {
 ....
}
```

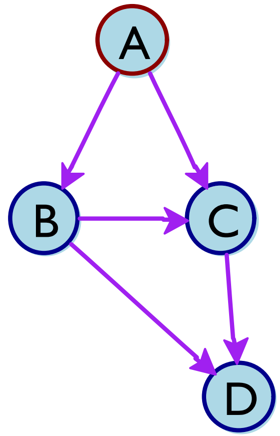
Designing Programs: Trees



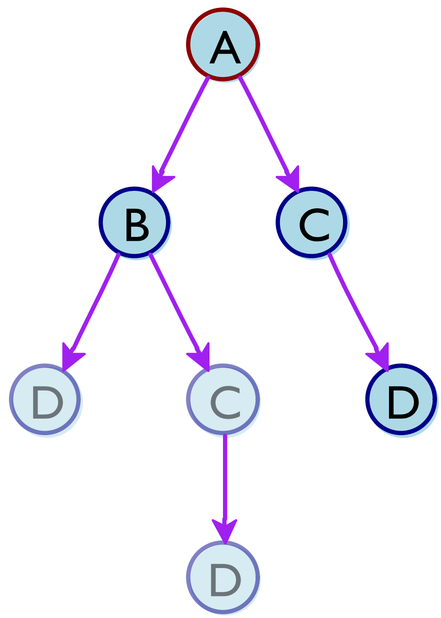
```
(define (F n)
  (cond
   [(empty? n) ...]
   [else ... (F (child1 n))
         ... (F (childN n)) ...]))
```

- Depth-first vs. breadth-first
- Might express recursion through a stack or queue

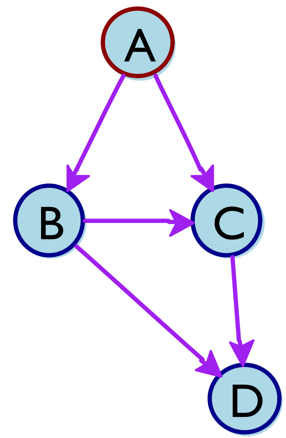
Designing Programs: DAGs



- Somtimes, treat a DAG as a tree

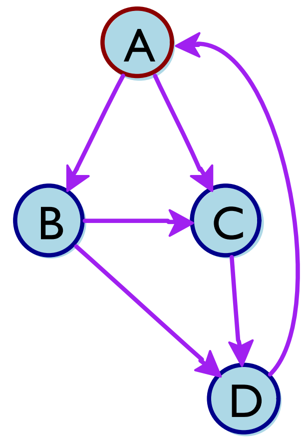


Designing Programs: DAGs



- Somtimes, treat a DAG as a graph...

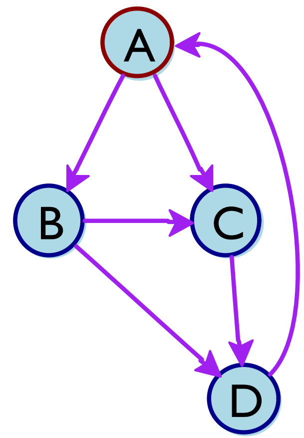
Designing Programs: Graphs



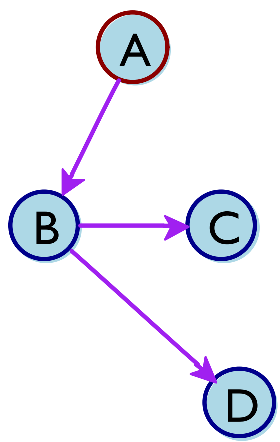
Like a tree, but accumulate seen

```
(define (F n)
  (cond
   [(seen? n) ...]
   [else (seen! n)
         (cond
          [(empty? n) ...]
          [else ... (F (child1 n))
                ... (F (childN n)) ...])]))
```

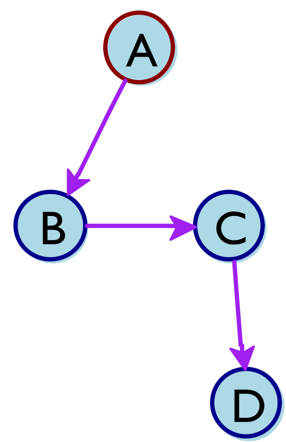
Designing Programs: Graphs



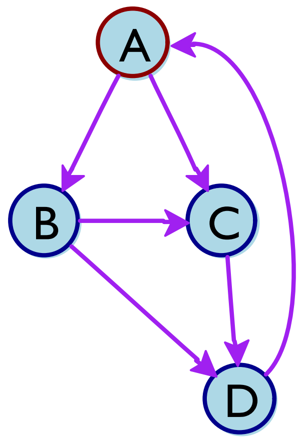
Depth-first:



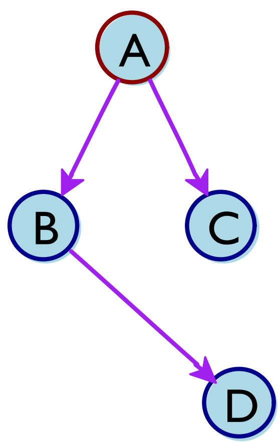
or



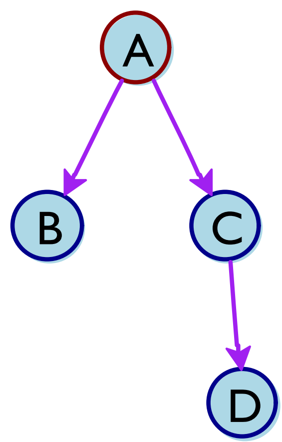
Designing Programs: Graphs



Breadth-first:

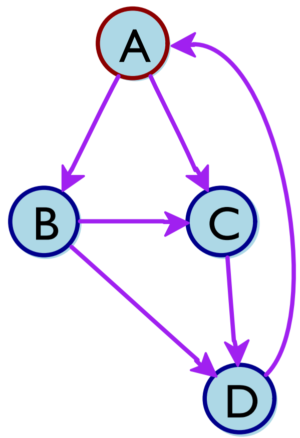


or

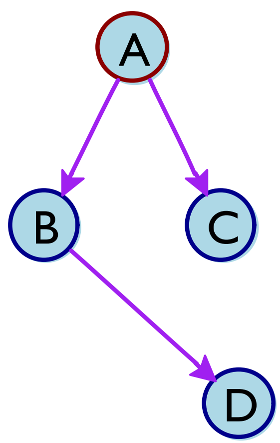


Classical Graph Algorithm

Find the shortest path to a node:

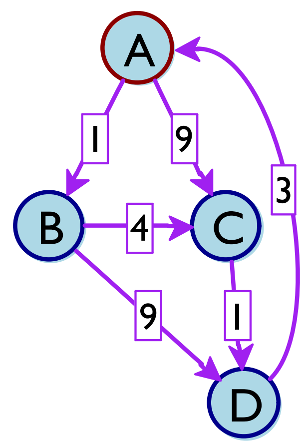


Solution: breadth-first search

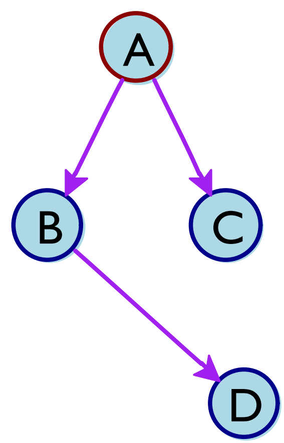


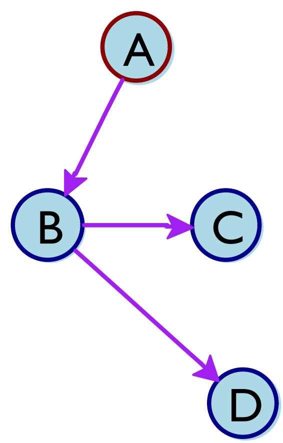
Classical Graph Algorithm

Find the shortest weighted path to a node:



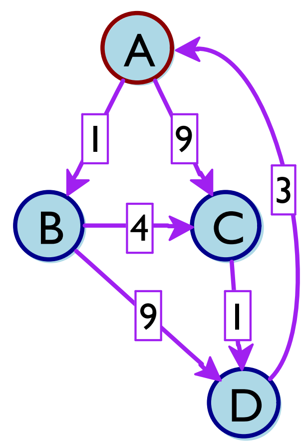
Neither breadth-first nor depth-first works





Classical Graph Algorithm

Find the shortest weighted path to a node:



Solution: use a ***priority queue***

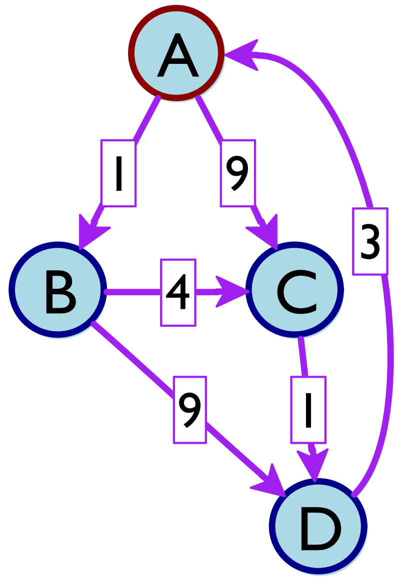
- Enqueue node with distance so far
- Dequeue node that has shortest distance so far

A priority queue gives us “closest-first”

- Instead of a queue (breadth-first)
- Instead of a stack (depth-first)

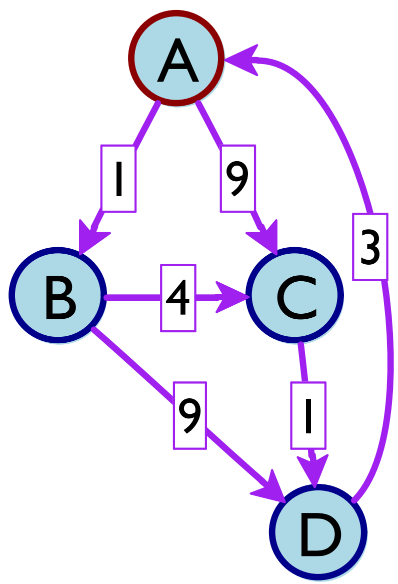
Shortest Weighted Path

0



Shortest Weighted Path

0

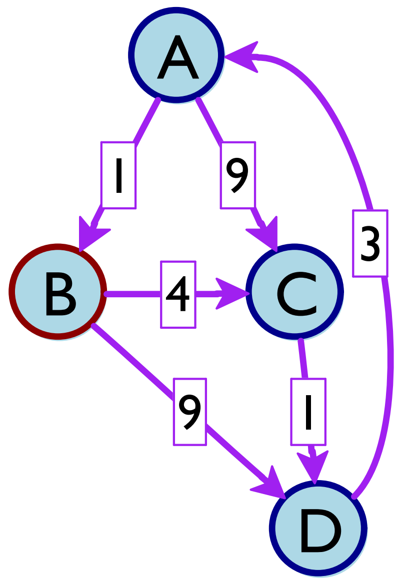


B 1

C 9

Shortest Weighted Path

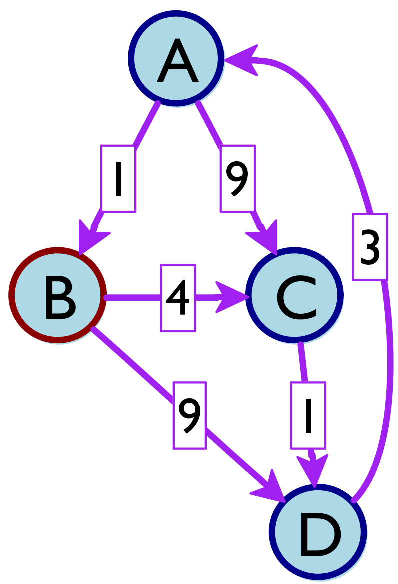
1



C 9

Shortest Weighted Path

1



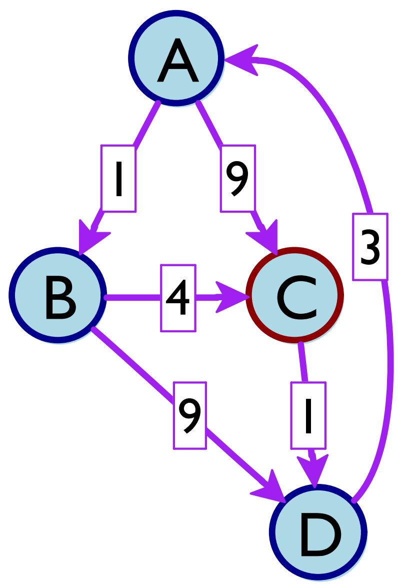
C 5

C 9

D 10

Shortest Weighted Path

5

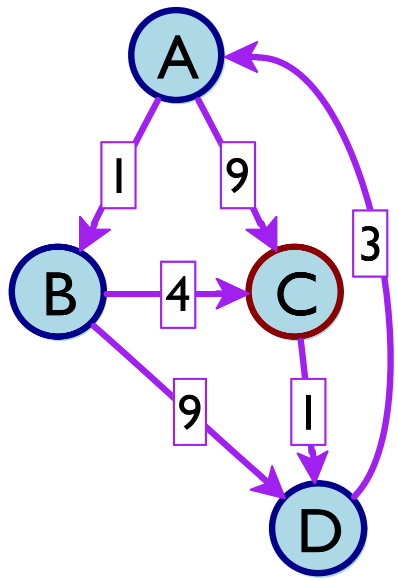


C **9**

D **10**

Shortest Weighted Path

5



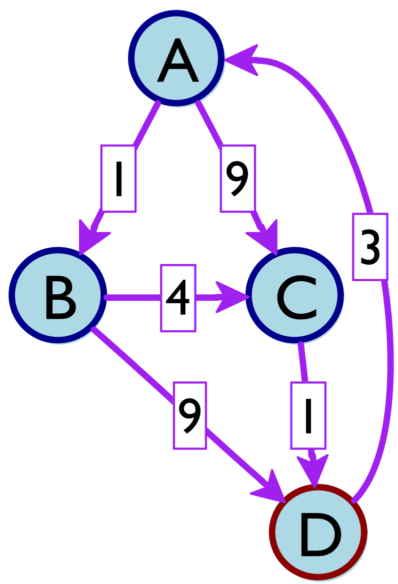
D 6

C 9

D 10

Shortest Weighted Path

6



C 9

D 10

Tracking Seen Nodes

Two common ways to track “already seen” nodes:

- Reserve space in the node for a mutable boolean
  - \+ Easy to implement (in C)
  - − Easy to pollute state
- Use a container
  - − More work to implement (in C)
  - \+ Avoids extra state