Project Summary

We propose to advance the state of the art in geometric modeling by constructing a set of new surface representations that are sufficiently general to solve a wide range of problems in geometric surface design, estimation, and analysis. Our assertion is that no single, representation is adequate to efficiently and accurately solve the myriad of computational and analytical problems that arise in modern applications. For this reason we propose the development of a framework that systematically combines parametric and implicit surface representations.

The problems addressed by this research are pervasive bottlenecks to progress and make usage difficult, so this work will have a broad impact on variety of fields that rely on CAD and geometric surface modeling. The proposed work will also have an impact on cartography, and will provide new tools for reconstructing, representing, and analyzing terrain data. The educational opportunities within this project also important; they will offer students a unique, but powerful blend of mathematics, computer science, and engineering.

Non-uniform B-splines (NURBs) are an efficient and accurate mechanism for modeling curved surfaces, and they are used extensively to solve problems in constrained approximation, analysis, computer aided design (CAD). Spline representations, however, can be inefficient, particularly for modeling operations—such as deformation, sweeping, or offseting—that entail local or global surface interactions. An implicit surface strategy provoking widespread interest is the method of level sets. It provides a framework for deformable implicit surfaces that are represented on a discrete grid. Level-set surfaces use implicit representations to represent local, free-form deformations and they are particularly effective at representing geometric surface evolutions. However, the disadvantages of this representation are significant. For instance, the discrete grid imposes a finite resolution, and the incremental shape changes can require burdensome compute times.

Recently, the literature has shown some interesting advantages for the use of hybrid implicit/particle-based representations of moving wave fronts [55, 51]. This early success suggests that other hybrid representations could be used to address the shortcomings of current representations used in CAD and topography. The purpose of this proposal is to link two representational strategies so that each overcomes the computational and representational shortcomings of the other. The proposed work will also study several fundamental problems in surface deformation and address applications of this technology to CAD and cartography. The goals of this project are:

Hybrid Representations and Computational Methodology: We will undertake the development of establishing systematic methods for hybrid spline/level-set representations. This will entail methods for having each representation *track* the other—so that either can be used as the driver.

Analysis of Topology and Singularities: This work will include an analysis of the singularities that can occur with different geometric operators acting on different representations. The proposed method will allow the representations to communicate information about singularities and topological changes.

Computer-Aided Design: We will apply these new methods to CAD problems in the context of our Alpha_1 research CAD environment. This strategy provides a test bed and will push our work toward practical algorithms. The use of Alpha_1 as a test bed will also drive the requirements for further capabilities in the underlying framework.

Computational Topography: We will apply level-set, parametric, and hybrid representations to several important problems in computational topography including terrain reconstruction and the analysis of the generic properties of geometrically-derived terrain features.

Collaborative Research: Hybrid Modeling for Geometric Design, Estimation, And Analysis

1 Introduction

1.1 Statement of Problem

The subareas of computer graphics (CAD, engineering analysis, cartography, and medical visualization) have historically relied on rather different surface representations in order to meet their distinct needs. Substantially different algorithmic and computational foundations have evolved and given rise to multiple, largely disparate surface formulations. Notwithstanding individual affinities for particular schemes, we assert that no single, representation is adequate to efficiently and accurately solve the myriad of computational and analytical problems that arise in modern applications. Because of differences in model representations and their resulting algorithmic and computational tools, researchers spend considerable effort trying to re-create and adapt techniques and methodologies developed in one representation to work with another.

To address this predicament, we propose to establish a unifying framework that compatibly, intimately, and structurally combines parametric and implicit surface representations. We will adapt and apply results in singularity theory to guide our approach to developing computational operators. To prove efficacy, we will apply this framework to several real application problems in CAD and computational cartography. The purpose of this proposal is to systematically merge two representational strategies, parametric and implicit surfaces, so that each overcomes the well-known computational and representational shortcomings of the other. By providing a consistent view of the models in a unified multivariate and volumetric representation, we will achieve the union of their complementary advantages. This advance will enable us to attack problems that are (nearly) intractable in one representation with powerful tools and techniques developed within another.

For this project, we will direct our research, for several specific reasons, toward a pair of representations, nonuniform rational B-splines (NURBs) and level-sets. First, they represent some highly important trends in the fields of modeling and simulation. Second, these two representations are complementary in modeling power. These two representations are also particularly conducive to applications singularity theory. Finally, this team of investigators is particularly well qualified to study the combination of NURBs and level sets in hte context of singularity theory. Because of the strong mathematical foundations of this work, the lessons learned from this project are expected to generalize to other representations as well. Furthermore, the focus on applications will provide solutions to specific problems of interest in CAD and cartography.

In computer-aided geometric design (CAD), computer graphics, computer animation, and computer-aided manufacturing, NURBs are ubiquitous in both commercially available packages and research testbeds. NURBs are pervasive partly because they are an efficient and accurate mechanism for modeling curved surfaces. They provide designers with nested function spaces that allow them to design hierarchically. In addition, their analytical and geometric properties make NURBs an intuitive basis for solving computational problems in design, geometric data fitting, and data reduction.

On the other hand, as with other parametric representations, there are certain types of modeling operations, (such as forming generalized sweep surfaces and offset surfaces, boolean combinations) that cannot be represented exactly within a NURBs space. While theoretically, they can be approximated arbitrarily closely, boolean combinations of NURBs surfaces suffer from inherent problems in implementation resulting from floating point arithmetic, ill conditioned model interactions, and robustness and consistency issues. Another difficult problem that requires significant manual intervention is embedding and modeling *feature curves* into surfaces, especially those that may bifurcate, like river basins or mountain ridges.

Another important approach to modeling, implicit surfaces provide the underlying representation of constructive solid geometry (CSG), but the geometric primitives of CSG are generally considered to be too restrictive for today's designers. Level sets are a method of representing implicit surfaces on a discrete 3D grid and computing surface motions by solving a partial differential equation (PDE) on the range of the model. The level-set representation and the associated computational methods provide a framework for computing local, free-form deformations with implicit surfaces. Level sets are particularly effective at representing geometric surface evolutions, where the deformation depends only on local surface shape. Because the representation is implicit, it can readily handle complex shapes and changing topologies.

Despite its power, the level-set representation comes with some significant drawbacks. For instance, the discrete grid imposes a finite resolution on the models, and stable numerical schemes for solving the associated PDEs introduce a *numerical diffusion* which further undermines their accuracy. Also solving the PDEs associated with incremental shape changes can require burdensome compute times. Furthermore, lacking an underlying global structure, surfaces represented as level sets cannot easily be used to carry other data, like structural relationships, mechanical properties or textures.

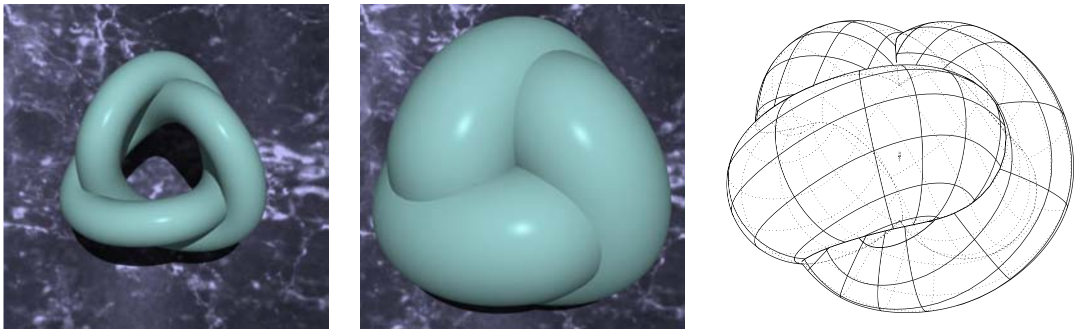
The problems discussed in previous paragraphs are not unique to these representations. They are general properties of parametric and implicit modeling schemes. For instance, virtually all parametric and mesh refinement schemes have difficulties handling offset surfaces and the associated self intersections. Likewise, no implicit modeling scheme has yet adequately addressed the problem of representing surface detail and discontinuities. Indeed, in this sense, the proposed research is quite general. As case in point of the various inadequacies of current surface representations consider the following examples.

Model self-intersections: When forming surfaces by the standard technique of sweeping one curve along another, one can inadvertantly produce a self interesection surface. Figure 1.a shows a model formed by sweeping a circle along a simple curve, and there are no self intersections. If a larger circle is swept along the same curve, as in Figure 1.b, numerous intersections result. Figure 2.a shows the intersection curves in the parameter space, and Figure 2.b shows the original surfaces inside the trimmed enlarged model. Recently there have been several results aimed at determining the self intersection curves for Bezier surfaces[2] and B-spline surfaces[75, 74], and one approach which carries it further to create a valid model[75, 74] Reconstructing the valid model is difficult and error prone, as is constructing a valid boolean model from surviving multiple surface pieces.

Feature curve placement: Another example of difficulty in parametric modeling involves is introducing, positioning, designing, and representing feature curves, particularly when those curves do not run along isoparameters. To generate NURBs representations from measured data it is necessary to move these discontinuities to fit the data. In many systems such a task requires significant manual intervention. Mesh-based methods can also have difficulty[70]. Figures 3 and 4 show different types of non-isoparametric features that can arise.

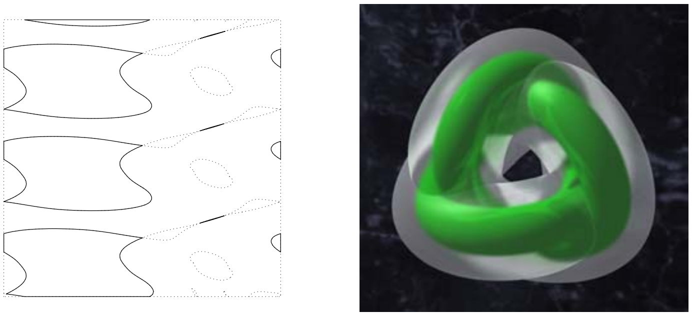
Finite resolution: One basic problem in using level sets to model surfaces is the finite resolution of the discrete grid. With careful implementations level sets can be manipulated to subgrid accuracy using some suitable interpolant [140]. Furthermore, researchers have explored adaptive methods [125]. Despite these techniques, the discrete grid is an inherent limitation in representing surfaces of high curvature. Figures 5(a)–(b) show the aliasing associated with the edges of cube when represented as the level set of a volume. With higher resolution, the accuracy of the model increases, but high-frequency artifacts near areas of high curvature remain.

Numerical Diffusion: Another problem with modeling surfaces as level sets is *numerical*



(a) (b) (c)

Figure 1: A swept tube: (a) no self-intersections, (b) multiple self intersections, (c) line drawing of (b).



(a) (b)

Figure 2: (a) Parametric space self intersection curves (dotted self-intersection curves bound regions totally interior to the object made by the trimmed regions, (b) Original tube interior to trimmed tube.

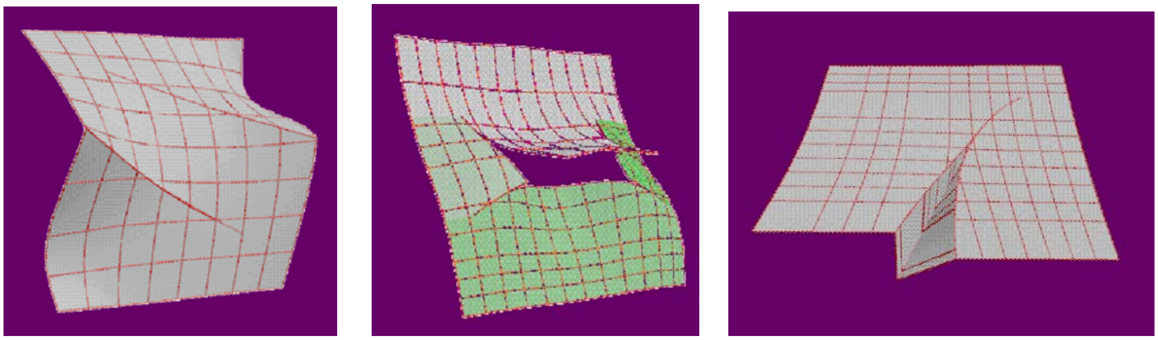
diffusion. Numerical methods for solving level-set PDEs incorporate a diffusion term that necessarily reduces high-frequency details in the surfaces as they evolve. Figures 5(c)–(d) show the effects of numerical diffusion on an advecting surface.

Rather than attempting to find the elusive universal representation with ideal computational tools, a more realistic plan is to explore carefully the structural implications that singularity theory brings to these two specific classes of representations and their modeling processes, and use these results to guide the development of our fundamentally hybrid approach.

1.2 Summary of Proposed Work

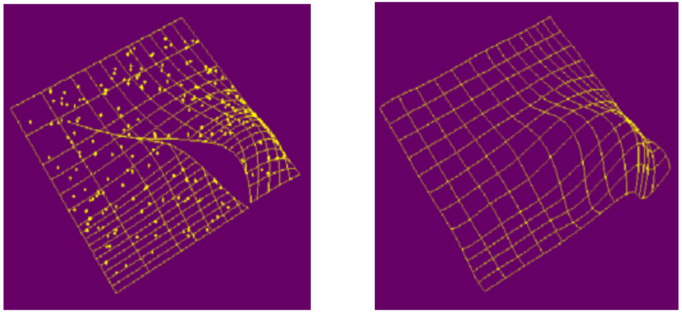
We propose to advance the state-of-the-art in geometric modeling by constructing a framework that mathematically combines parametric and implicit surface representations. The goal of this framework is to develop a hybrid representation for solving modeling problems that neither individual representation solves completely or well on its own. Our specific research goals for this project are as follows.

Mathematical Foundations: We will develop the mathematical underpinnings and algorithms for the proposed hybrid modeling approach and create appropriate modeling operators. Rather than merely translated models, the two representations will be linked as they undergo transformations simultaneously. Each one will inform the other of changes in its state, by the dectection and analysis



(a) (b) (c)

Figure 3: a..... b.



(a) (b)

Figure 4: a......b

of singularities.

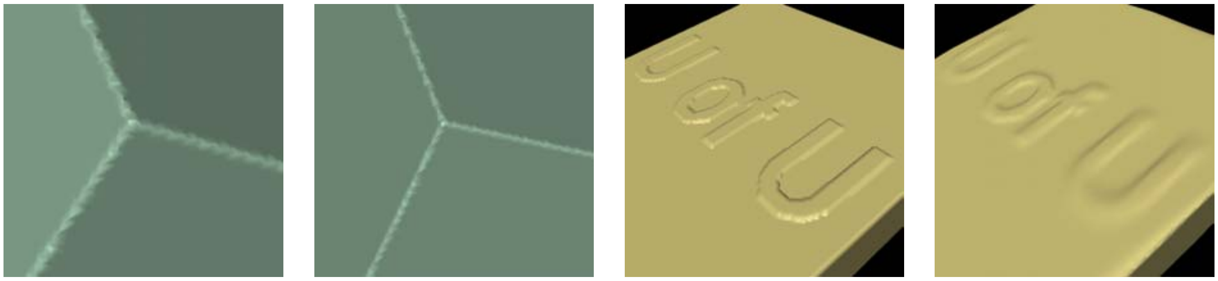
Computational Methodology: A significant challenge of this work is making this hybrid representation practical. This entails efficient algorithms and implementations.

Applications: The proposed work will focus on two application areas, CAD and computational cartography.

Driving Problems: Early in this project we will define a handful of driving problems that link the theory and algorithms work with the applications. These problems, compelling, worthy challenges in their own right, will also become part of the overall framework we are developing.

Knowledge Transfer and Utilization: The proposed work has broad implications for a variety of fields. Besides the conventional academic venues of publishing and speaking this team will pursue two other means of disseminating this work. The first is through the educational process, where we will establish a unique combination of numerical analysis, computer science, and mathematics. The second avenue is through industrial contacts and open source software.

The team assembled to lead this research is unique. James Damon is a leading researcher in singularity theory and has a track record in solving significant problems related to those in the proposal. Rich Riesenfeld and Elaine Cohen are world reknown for their fundamental contributions to the field of spline modeling and computer-aided manufacturing. Ross Whitaker has an extensive record in surface reconstruction and other applications of level sets and is one of only a few researchers who has systematically examined the question of terrain reconstruction from noisy LIDAR data. Our team has experience working together (smaller subgroups) and has a strong plan for maintaining the collaborative nature of this project.



(a) (b) (c) (c)

Figure 5: Aliasing artifacts on high curvature surfaces (a) with resolution of 128 grid points and (b) with a resolution of 256 grid points. A level-set surface (c) moving under an advection through a 10 degree rotation looses signficant detail (d) due to numerical diffusion.

1.3 Motivation and Impact

We see a significant research opportunity to advance the state-of-the-art in modeling by creating a fundamental combination of two widely used representations and introducing ideas from singularity theory to take geometric modeling in a new and important direction. Moreover, we believe the lessons learned will be more generally applicable to creating other hybrid modeling strategies. We believe that bringing the concepts of singularity theory to bear on geometric modeling will have significant impact in dealing with difficult problems concerning the behavior of common geometric operations.

Science and Engineering: Because there are a relatively number of commercial systems that rely on NURBs, and because there is large base of NURBs-modeling expertise, our proposal to inject this modeling method with new functionality will improve the power and productivity of such systems. This work will also impact the way people use B-splines to solve approximation problems, and it will provide new tools for solving problems entailing differential equations over curves and surfaces, including geological and cartographic reconstructions. Level sets are now used for in scientific computation, image processing, and visualization. Successfully forming the hybrid representation and developing appropriate and efficient methodology will provide new level set algorithms, making them more accurate and efficient.

The Utah-UNC team has track record of work that impacts the military and industrial communities, and we experience in transfering technology to the commmunity at large. Our software infrastructure for manipulating level sets, *VISPack*, is open source, and is available by on the web at *http:/www.cs.utah.edu/∼whitaker*. Furthermore, the Utah team is part of the *Insight Project*, an NIH sponsored project to develop an open source toolkit for medical image processing (see *http:/www.itk.org*), which includes deformable surface models and level-set solvers.

Our research on feature-based NURBs modeling, process planning, and manufacturing has become a commercial product with significant military use. The main tech-transfer vehicle for the CAD related research has been Engineering Geometry Systems (EGS), a spin-off company founded by Cohen and Riesenfeld (see *http:/www.featurecam.com*) to facilitate rapid adoption of new research. This company, which has licensed technology developed at Utah, markets world-wide and services both Fortune 500 companies and smaller shops. Riesenfeld is the Chairman of the Board of Directors of EGS, and they will continue as a conduit for rapid commericalization of CAD technology developed at Utah.

Cartography and Defense: The research results from this project would be immediately useful for two ongoing research projects sponsored by the Department of Defense (DoD). The first is the *Virtual Parts Engineering Research Center* which is sponsored by the Army Research Office. This project focuses on unifying methodologies for rapid recovery, redesign and manufacture of small numbers of mechanical assemblies . This project provides driving problems and another conduit for disseminating the results of the proposed work.

The other project is entitled *A Statistical Framework for Terrain Reconstruction*, and it is sponsored by the Office of Naval Research. This project addresses the question of building 3D surface models from noisy LIDAR data taken from airborne platforms. This project incorporates researchers from the Naval Air Warefare Center at China Lake, California. The proposed work will provide us with new tools for manipulating and analyzing height fields and terrain models. Furthermore, this ONR project provides us with direct access to data sets and hands-on experience with cartographic problems—we believe this experience be essential in pursuing the proposed research.

Educational Impact: This research provides the opportunity to work with students across broad array of disciplines. Based on other already ongoing collaborative research, Professor Damon at UNC is initiating a new course on "Singularity Theory and Computer Imaging" in spring 2003, which will be attended by groups from both the Computer Science and Mathematics Departments. As our collaboration starts yielding results, we hope to see course related to singularity theory in modeling. Via our NSF-sponsored televideo connection, UNC and Utah can have regular research meetings and involve students with faculty from both sites.

Professor Cohen has served as a mentor in the CRA distributed research mentor program, mentoring 5 undergraduate women during the summers of 2001-2002. We are pleased that several are planning to apply to graduate school, and several have developed increased interest in mathematics. During the academic year, Cohen, Riesenfeld, and Whitaker all have had undergraduate interns working in their labs, and we will involve undergraduate interns in this research.

Finally, our outreach has extended to high school students. We have run summer programs for high school students for more than a dozen years on graphics, modeling, and expert systems. Singularity theory, level sets, and NURBs are not high school math material, but experience with the proposed computational tools will convey the intuitive aspects of this work.

2 Technical Background and Related Work

The proposed work spans a large body of work in several different fields of mathmatics, computer science, and engineering. This section describes the technical and historical context of the proposed work and the Investigators' own work in these fields.

2.1 Relevant aspects of NURBs

Since Riesenfeld[121, 122] first proposed using nonuniform B-spline surfaces to solve geometric problems in CAD, their inherently attractive characteristics have led to their widespread use, culminating in their position as *de facto* standard in the CAD industry. Today, nearly all contemporary commercial modeling environments and CAD systems (including Pro-Engineer, Ideas, ParaSolids, AutoCad, Rhino, SolidDesign, and FeatureCAM) employ this surface representation.

NURBs have a number of attributes which make them attractive as a modeling representation (see [123, 42, 21], among others). All spline spaces contain the space of polynomials, shape mimicking B-splines provide the designer a small number of descriptive handles (e.g., control vertices), which allow easy modification of surface properties and serve as parameters for automated design and optimization operators. Smoothness is ensured by the representation. The relationship of a

B-spline curve or surface to its control net exhibits convex hull, variation diminishing, and total positivity behavior - all conducive for creating efficient computational algorithm. B-splines can act as a low pass data fitting filter, and so act to smooth data. B-spline basis functions have local support enabling edits whose impact is isolated from the rest of the model. Efficient schemes for computing differential properties as well as surface properties have been developed. Elber[46] introduced hybrid symbolic and numeric computation for NURBs surface analysis. Moreover, efficient methods of refinement facilitate modifications at many scales[19, 98] and a multiresolution approach to design and analysis. There are even uniform and nonuniform spline wavelets[17, 99] to support modeling, analysis, and rendering for those applications conducive to a wavelet approach. For applications where fitting is key, a variety of interpolation and approximation techniques are available to fit B-splines to data.

While tensor product parametric NURBs surfaces are best suited to modeling rectangular surfaces, there are generalizations that allow that domain to take a more general shape. Relying on their critical refinement algorithms[20], a specific sub-class of the box-spline representation, an intrinsic multivariate spline over a hexagonal grid, has been used to derive subdivision surface templates and for their analysis, for instance[95, 148, 70]. A different approach, called *torn surfaces*[49, 50], allows almost arbitrary creases, surface discontinuities, and feature curves to be embedded in an adapted tensor product formulation. Developed for problems in multiple surface problem areas, including thin plate deformations (failure simulation), stamping, and geology (non-manifold layering), this representation will be used in our research.

In 1980, Cohen *et al.*[19] proposed that the newly developed Oslo Algorithm[19] could be used to support boolean operations on models bounded by NURBs surfaces by subdivision in combination with numerical root finding, to create *trimmed* surfaces, and to analyze model topology. Even though research using multiple approaches to create robust algorithms for surface-surface intersection and correct topology has been active since that time (for example, [136, 5, 52, 67, 76, 93, 117, 146, 89]), there are still significant research questions and formidable implementation problems including robustness, consistency, and correctness itself. Subdivision surface booleans require a similar analysis and topology determination[6], and so would be prone to the same problems in complex boolean operations. In spite of these problems, the trimmed NURBs representation is widely used to represent boolean models from sculptured surfaces. A related problem, surface self-intersection, arises as a result of many common modeling operations, such as sweeps, offsets, approximation of poorly parametrized or poorly conditioned data, and deformations. Surface self-intersections can be characterized as local and global types. While some approaches to solving the problem just insert the same surface in a two surface boolean operation, that approach has instabilities when the self-intersection curve is a single loop, see Figure 6. Ho[75] showed that the corresponding parts of the curve meet at surface singularities, and standard methods fail near the singularities.

Deficiencies in geometric computations discussed above can be made more effective through the hybridization with level sets and singularity theory. In a complementary way, NURBs and singularity theory will help in overcoming deficiencies in level sets.

2.2 Relevant Aspects of Level Sets

In 1988 Osher and Sethian [115] proposed the method of level sets for implicitly modeling moving interfaces. The method represents the interface as the set of points

$$\mathcal{S} = \{\vec{x} | \phi(\vec{x}, t) = k\} \tag{1}$$

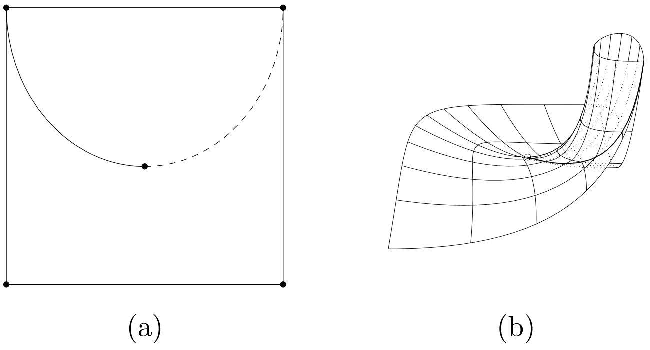


Figure 6: A self intersecting surface blending to smooth (a) surface (b) parameter space self-intersection curve.

where $\phi : \Re^3 \times \Re^+ \mapsto \Re$ for 3D surfaces and k is generally chosen as 0. If we let $\vec{v}(\vec{x})$ be the velocity of a point $\vec{x}$ on the interface, and we insist that the $\mathcal{S}$ remain the kth level set over time, we have

$$\frac{d\phi(\vec{x},t)}{dt} = \nabla\phi \cdot \vec{v} + \phi_t = 0 \ \Rightarrow \ \phi_t = -\nabla\phi \cdot \vec{v}, \tag{2}$$

where the subscript represents a partial derivative and $\nabla\phi$ is the vector of partial derivatives with respect to the spatial parameters.

In this way, the incremental motion of an interface is computed as the solution of a 3D, nonlinear PDE. Solving these PDEs on a grid using finite differences requires careful attention to the discrete approximation of the gradient of ϕ. Osher and Sethian proposed the *up-wind* scheme as a method for computing the viscosity solution to (2). The computational burden associated with solving these PDEs is considerable, and several authors [1, 144] have proposed numerical schemes that track the moving wavefront and perform updates only where necessary. Over the last decade, researchers have demonstrated effective uses of level-set methods in a variety of fields ranging from geophysics to medical imaging and computer graphics, e.g. [145, 101, 125, 53, 55, 107]. In this section we give a short dicussion the literature that is particularly relevant to this proposal.

In CAD, Kimmel and Bruckstein [91] propose the use of level sets to compute offset surfaces and Kimmel *et al.* describe a level-set mechanism for computing shortest distances on surfaces. Whitaker and Breen [140] demonstrate the use of level sets for blending and filleting. Whitaker [144] describes the use of level sets for 3D reconstruction from LIDAR data and presents a technology for fitting level-set models to sub-grid accuracy. More recently, Museth *et al.* describing a level-set system for surface design and editing, that includes a toolbox of smoothing, blending, and texturing operators. Despite this progress, level-sets have not made their way into mainstream CAD applications for the reasons discussed in previous sections.

The numerical diffusion in discrete level-set methods *has* motivated other hybrid modeling approaches. In particular, Fedkiw *et al.*[55, 51] have proposed methods for fluid simulations that combine level sets and particle systems. This hybrid strategy helps to overcome the numerical diffusion problem in level sets; the particles *correct* the level sets as they move. This particle/level-set hybrid suggests that hybrid schemes can be used to overcome other surface modeling short comings, and it is part of the motivation for the proposed work.

2.3 Relevant Aspects Of Singularity Theory

To understand the singularities resulting from some form of level set flow or the intersection of NURB surfaces, we will use methods from singularity theory. One major part of singularity theory concerns the properties of smooth mappings between smooth spaces (manifolds) $f : M \to N$. This includes the classification of the local forms for *generic mappings* and their properties, conditions for the stability of both local and global properties under small perturbations, and the characterization of generic transitions that occur as parameters vary.

From its beginning in the early 60s as a result of the pioneering work of Rene Thom, John Mather, and V. I. Arnold, singularity theory has evolved into a rich subject with many practical applications. It has expanded into a large body of results which solve the preceding problems in situations well beyond the above abstract formulation by allowing numerous additional constraints. As a result it has been applied (in literally hundreds of papers) to questions in bifurcation theory, pattern formation, Hamiltonian mechanics, PDEs, differential and algebraic geometry, and the like.

In recent years singularity theory has become an especially useful tool for various problems in computer imaging such as: i) determining the generic properties for Blum medial axis and its generalization; ii) the symmetry set and relating these to shapes of objects, e.g. [147, 110, 14]; iii) determining the changes which apparent contours of objects undergo under viewer movement and deducing geometric properties of objects in images [92, 57, 3]; iv)determining geometric properties of objects in medical images via Gaussian blurring of the image, e. g. [30, 34]; v) deducing the smoothness and geometric properties of boundaries of objects from data on the Blum medial axis and its generalizations, e.g. [27, 28].

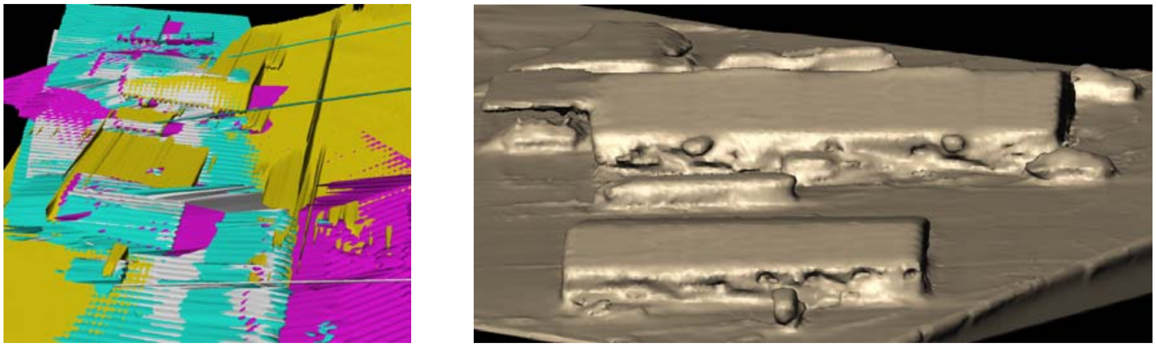
Researchers have studied these phenomenon without the use of singularity information. However, the results obtained from singularity theory usually provide a deeper understanding of such problems and have helped to explain algorithms that did not behave as expected, e.g. [31, 36, 28].

For example, the Blum medial axis can be described as the *Maxwell set* (where multiple minima occur with the same minimum value) for the distance to the boundary function [110]. The knowledge of the generic properties for such functions yields concrete criteria for the properties of the Blum medial axis and how this can be expected to change under variations of the boundary surface (see also [60]). Alternately, the medial axis can also be described as the shock set for the grassfire flow [90]. Singular properties of shock sets for such flows have also been analyzed using singularity theory, e.g. [62, 10, 79]. Furthermore, using a backwards version of this flow, the *radial flow*, geometric data on the shock set can be transferred back to yield geometric information about the boundary [27]. We foresee a similar use of singularity theory for understanding singularities appearing in level set flow, NURB surfaces, and geometric features for terrain analysis.

2.4 Reconstruction and Terrain Modeling

The problem of reconstructing terrain models is often addressed in the field of remote sensing in combination with photogrammetry and stereo vision [71, 61]. Of particular interest is the use of range data, developed from stereo images, to build terrain models, usually of buildings and other man-made structures, e.g. [23, 78, 102]. Such work typically addresses the stereo matching problem (from distances where occlusions are not significant issue) and the problem of fitting domain-specific models, such as buildings or ravines [73, 137], to monocular or stereo images. The proposed algorithm deals instead with low-level reconstruction, and does not make domain-specific assumptions.

Several authors have addressed the specific issue of building generic terrain models from laser-range data. For instance, Keon and Kanade [94] describe a robot-based system that registers and integrates range images by first scan converting onto the ground-plane coordinate system. They model uncertainty and perform view averaging in the model space rather than the sensor space. Huber and Hebert [77] propose a similar system that incorporates a surface zippering approach that does not include a sensor noise model. Both of these methods require explicitly detecting



(a) (b)

Figure 7: (a) Four noisy LIDAR scans taken from an airborne platform are registered using the method in [142]. (b) These four scans are combined in a Bayesian framework by fitting 3D surface models.

occluding boundaries, a difficult edge detection task. This is not required for the method proposed in this paper. Recently, researchers at the U.S. National Institute of Standards and Technology have proposed using range data to monitor construction sites [16]. They develop surfaces from triangulations of 3D point clouds—an approach that is very sensitive to sample spacing and does not systematically address noise.

In previous work [144], the PIs pose the problem of reconstructing the shapes of 3D objects from multiple range images within a statistical (Bayesian) framework, which results in an optimization problem. That formulation is rather general, and does not rely on any specific surface representation. An iterative strategy for the optimization results in a sequence of surface deformations. That work shows full 3D reconstructions using the method of *level-sets*, an implicit surface representation that relies on a discrete, 3D grid. More recently Whitaker and Juarez-Valdes [143] have demonstrated the use of deformable height fields in order to build 3D terrain models from noisy LIDAR data. This work stands to benefit from some recent advances in 3D deformable models, which allow us to smooth surfaces while retaining sharp corners [134, 132].

The reconstruction and terrain modeling work at Utah takes place in the context of two ongoing Department of Defense projects. The first is through the Office of Naval Research (N000140110033). That project, which entails a colloboration with the Naval Air Warefare Center (NavAir) at China Lake, specifically addresses the questions of terrain reconstruction and analysis. Some results from that project [1] are shown in Fig. 7. The other project is with the Army Research Office ([**?**]), and it addresses, among other things, the use of scanned data for reverse engineering of legacy components in mechanical systems. Preliminary results from that project are shown in Fig. 8.

3 Proposed Work

The proposed work addresses two open general questions pertaining the use of geometric models for surface modeling and terrain analysis. The first is "How do we achieve robustness for certain classes of prevalent geometric modeling operations?" Many of the difficulties arise at singularities. Approaches to solving these problems without appropriate theory are usually based on empirical methods, most often by *tweaking* the model or the process that creates the model. The operators are frequently complex to use and can require manual intervention. At best it leads to a computational

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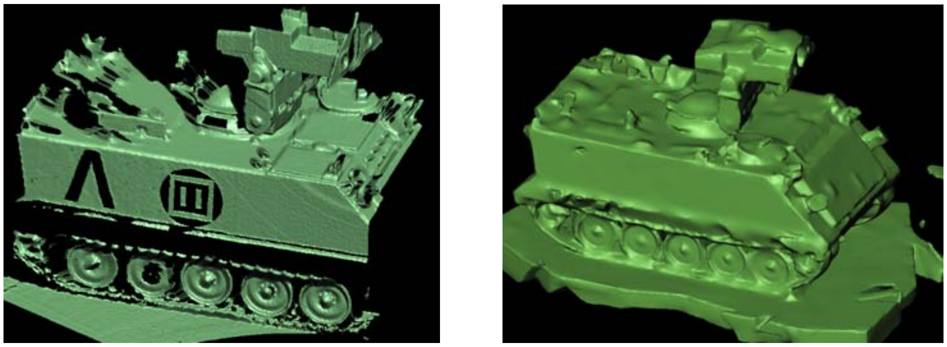
[1] Terrain data courtesy of the CMRTR Ladar Development Team at the Naval Air Warfare Center at China Lake, California.



(a) (b)

Figure 8: (a) One of eight LIDAR scans of an armored vehicle. (b) Scans are combined through a surface-fitting process to build 3D level-set models.

result on a per item basis. At worst, it fails to obtain a reasonable and consistent model. Then, using the mathematical approaches we derive in our research from the first question, we address "What kinds of modeling operations and operators can be automated automated" with these tools, and "Can we we build/analyze terrain models using the resulting representation and tools that offer a significant advantage to the terrain modeling comunity over other approaches?" In the sections that follow, we present specfic problems that must be solved in the creation of the representation, the research challenges, and our proposed approach.

3.1 Research Challenges and Approaches

3.1.1 Mathematical Foundations

This part of the proposed work concerns the development of the mathematical underpinnings of the hybrid representation and expressions of the appropriate modeling operators.

Because translation (back and forth) between nonequivalent representations is a lossy process, we do not consider it an appropriate strategy for the proposed hybridizations. Our stategy is therefore as follows. Given a surface model in one representation or *format*, and an operation which requires the complimentary format, we would like to build that other representation on the fly, perhaps locally, and use information from that other format to guide the operation. This requires several underlying capabilities: generating the surface model representation, tracking the hybrid model, detecting singularities, and communicating between the formats.

Surface Model Generation: An important capability is generating models of one format from the other. Generating volumes from NURBs is not difficult, but the reverse process, automatically deriving a suitable (trimmed) NURBs model from a level set, is largely unsolved. The barriers to this include the complex topologies that can occur in level set models and partitioning each connected component into appropriately bounded data sets for fitting with NURBs to create trimmed models. We also plan to study the possibility of generating a local *sister* representation on the fly, only as needed. This effort will necessitate creating methods to solve the level set to NURBs transformation with local transformations.

Hybrid Model Tracking: During geometric operations in which we maintain both representations, they will need to be kept consistent during the geometric operation. This may require us to create and update correspondences between the implicit representation and the parameterization or control points of the NURBs surface. This will be particularly difficult for the cases in which a level set deformation leads to changes in topology of the model. The literature [124, 25] demonstrates schemes that embed the control points for the NURBs into another model and use their motion in the deformation to keep an approximate location. Refinement[19, 98] can be also used for better tracking and add degrees of freedom (DOF) to the NURBs model as needed, with accuracy and speed tradeoffs determining the level detail.

Detecting Singularities: We believe that singularities in surface shape during modeling operations will signal the events for which the two formats must interact. Furthermore, we expect different kinds of singularities in each of the formats. The challenge will be to detect them, classify them, and formulate the appropriate actions in each of the sister formats.

A crucial question for level sets is the appearance of singularities as the flow progesses. The investigation will proceed in two directions. One approach is to analyze the results from the literature concerning singularities of hyperbolic equations (there appear to be few) and adapt those results to this problem. Another approach is to apply methods we have developed to analyze generic singularities appearing in solutions to certain types of PDE's, including the use of special equivalences to capture special properties not treated by ordinary singularity theory [31, 30, 34]. For instance, Damon is currently supervising a Master's student who is applying the general results of Mather for classifying the generic singularities appearing in the shock set for the *grassfire flow* [90], and we will look at the behavior of these solutions with an additional curvature term. Concrete results exist for planar curves [58, 59, 68]; the evolution of curves under curvature flow do not develop singularities. However, we know from examples that the case of (general) surfaces evolving under mean curvature flow is significantly more complicated, and questions remain.

There are currently no direct results on detecting singularities in NURBs modeling operations, and this stands as an important problem for this project. However, offset surfaces, important for many NURBs modeling and manufacturing operations, fall into the class of *skeletal sets*. Recent research into methods for determining singularities for boundary surfaces with associated skeletal structures provide us with an approach for this important class of problems [39, 40, 38]. In place of normal vectors of fixed length, we can also study the behavior of surfaces moving with motions defined by general space/time varying vector fields, which is a generalization of the offset surface. We have done this by introducing [27] *shape operators* associated with the vector field. The open question in this project is how to adapt these tools to derive an analogous set of results for NURBs.

Communication: To keep the two formats synchronized requires significant communication between the two surface formats. The hybrid representation should help make the self-intersection process more robust, but requires more than just tracking correspondences. The question is how to encode important information (e.g. the appearance of a singularity) in one format such that we can use it in the other format to make the appropriate changes to the model. This will be done *as* the front evolves, and so it must be done efficiently.

3.1.2 Computational Methodology:

A significant challenge of this work is making this hybrid representation practical. This entails efficient algorithms and implementations. The level sets rely on a discrete grid, which raises the trade-offs between compute time and resolution. Likewise, the detection of singularities will require discrete decisions. In the case of NURBs, we will need to represent and manipulate geometric structures that model discontinuities. The process by which we fit such discontinuities, for instance to terrain data, is still uknown. This part of the work addresses the significant numerical and computational issues surrounding the proposed work.

3.1.3 Driving Problems

During the early phases of this project we will define a handful of driving problems that link the theory and algorithms work with the applications. These problems are meant to be compelling, worthy challenges in their own right, but also help us to formulate the relevant questions for the fundamental aspects of this work. Candiates problems include but are not limited to:

- Studing of singularities of level-set motions, NURBs deformations, and how they can be detected in a computationally tractable way,
- Using the level-set information to find/disambiguate singularities and avoid inconsistencies in parametric models. Tracking intersection curves and obtaining correct topology can be arbitrarily difficult, and special cases are ill conditioned. We hope the level set format will allow us to *jump over* such special cases to maintain topological consistency.
- Using NURBs models to keep fine detail during level-set deformations. Because of the grid based approach in level sets, numerical diffusion occurs as the fronts move. NURBs attributes may be a mechanism to keep the fine detail.
- Using of NURBs model information to embed sharp features in level set models.

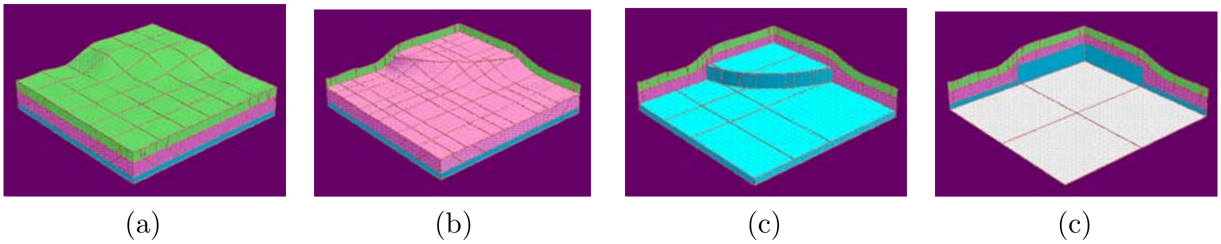
3.1.4 Applications

The proposed work will focus on two application areas, CAD and computational cartography. In the case of CAD, we intend over the long term to provide new sets of tools to designers that give them the *feeling* of NURBs, but with significantly more power including the ability to change topolgy, automatically resolve self intersections, etc. The driving problems solved in 3.1.3 will serve as the computational engines for the initial modeling operators.

The other application area is that of computational cartography. In this case we intend to similarly apply to combination of discrete and parametric representations for reconstruction (e.g. from lidar data) and analysis.

Watershed and runoff problems seem to involve dynamical system question for the *gradient flow*, see e.g. [128], whereby small changes in topography can lead to signficiant changes in the ridges which separate the cachement basins. Damon has methods for understanding the topography of a surface given as the graph of a function in terms of another structure, the *relative critical set*, which may be applicable. This method has the advantage that it is structurally stable—sufficiently small perturbations of the function will slightly perturb the relative critical set, but will not alter its properties. The relative critical set overcomes a key drawback of the notion of height ridge introduced by Pizer and Eberly [44, 118], which generically yields disjoint pieces of curves. These curves are only part of the complete *relative critical set structure.* The generic properties of this structure and the generic transitions in one parameter families has been worked out in dimensions 2 and 3 [34, 36], and in high dimensions[105, 88]. This is applicable to topography because a graph of a function in two variables generates a network of curves labelled by one of four types: ridge, valley, r-connector and v- connector. Our previous work gives us an precise understanding of which combinations of these structures are generic.

The above approach will enable characterization of the terrain. To fit surfaces to this characterized data, we will use the hybrid representation. The approach will be based on our previous individual work in the two separate areas. In particular, we will use the work of Whitaker and Juarex-Valdes[143] introduced in Section 2.4 and torn surface generalization of NURBs[49] introduced in Section 2.1 as our sister formats and operations on them as the terrain fitting operators. This will allow us to introduce arbitrary creases, valleys, and ridges into the representation. Figure 9 show modeling an underlying upthrust in the topograhy with the torn representation, while



(a) (b) (c) (c)

Figure 9: ,,,

Figure 4.b showed data interpolation with a standard formulation and with the torn formulation in Figure 4.a.

3.2 Plan of Work

3.2.1 Project Management

Even though the team is geographically distributed, we expect to work together quite closely. We have already set up two mechanisms to accomplish this. First, in the first year, Professor Damon expects to make three working visits to the Utah team, each one of a week duration, and later two working visits. Second, we plan to meet using an existing televideo link between the School of Computing (at the University of Utah) and the UNC-CH Department of Computer Science. This link was developed while both sites were members of the NSF STC for Computer Graphics and Scientific Visualization with support from the STC and a collaborative NSF MRI grant (Number [**?**]). Further, as appropriate, Professor Damon can serve on PhD committees for students at Utah, with the televideo facility can enabling frequent meetings between such students and Professor Damon. Finally, as the collaboration progresses, we expect to have a joint seminar via the televideo facility in which both mathematical and engineering results are presented.

3.2.2 Milestones

Year 1:
Driving Problems: boolean union of geometric models, addressing numberical diffusion.
Mathematical Foundations: generating models of one format from the other, coordinated tracking, applications of singularity theory to level-set PDE's.
Applications: select basic CAD problem (e.g. swept model), embedding torn surface and level set representations to terrain modeling.
Year 2:
Driving Problems: augmenting level-set detail, NURBS/level-set refinement/DOF
Mathematical Foundations: singularities on NURBs and 3-D level sets with curvature. communicating when/how to refine and associating new formats
Computational Methodology: effective tracking and communications
Applications: demonstrate year1 results on real terrain data and basic CAD operations
Year 3:
Driving Problems: self-intersections and topology changes
Mathematical Foundations: singularities and topological analysis of terrain data
Computational Methodology: effective iterative methods on data fitting problems
Applications: demonstrate torn/level hybrid on terrain data fitting, additional CAD operations

4 Relevant Results from Prior NSF Support

- **(A)** CAREER: A Statistical Framework for Reconstructing 3D Manifolds From Range Data (CCR0092065, P.I.—Whitaker)
- **(B)** Interactive Level-Set Modeling for Visualization of Biological Volume Data Sets (ACI0089915, P.I.—Whitaker)
- **(C)** NSF Science and Technology Center for Computer Graphics and Scientific Visualization. (EIA8929219, P.I.—Riesenfield, 2/91–7/02)
- **(D)** Topology of Nonisolated Singularities and Scale-based Geometry (DMS0103862, P.I. — Damon, 7/01–6/04)
- **(E)** Topology of Nonisolated Singularities and the Geometry of Functions (DMS-9803467, P.I. — Damon, 8/98–8/01)
- **(F)** Topological Properties of Singularities and Solutions of Nonlinear Equations (DMS-9400930, P.I. — Damon, 7/94–12/97)

Funding from (A), which is ongoing, supports development of the theoretical foundations for surface estimation. That work addresses the question of how to associate noisy measurements with surface shape and to solve such problems in a variational (Bayesian) framework. This framework has, so far, produced published results that include: a Bayesian formulation for estimating 2.5D (height functions) and 3D, free-form surface models from sets of range images, a maximum likelihood formulation for estimating shape parameters of 3D surface models from sets of range images, and a Bayesian formulation for estimating 3D surface models from limited or sparse-angle tomographic data. Relevant publications are [141, 143, 69, 134, 133, 132].

Project (B), which will end this year, supports the development of interactive level-set surface models. The emphasis of that project is user interaction and parallel computation with applications to biological data sets, in collaboration with biologists at UC San Diego and Cal Tech. Project (B) has led to new tools for users to directly manipulating surface shapes, as well as applications of these ideas to full-scale volume data sets. Relevant publications are [45, 141, 13, 12, 107, 108].

The results from (C), the STC, are numerrous. Since NSF is concerned with recent publications, we include only those publications occurring since 1997 by topic: assemblies [43], collaborative design and fabrication [96, 24, 4, 81, 120, 119, 100, 97, 97], constraints in modeling [113, 111, 80, 81, 106], haptics [138, 112, 114, 85, 83, 139], minimum distance algorithms [86, 84], modeling and analysis [103, 22, 47, 9, 75, 50, 72], non-photorealistic rendering [15, 127, 87, 66, 65, 63, 64], process planning and fabrication [7, 8, 48, 75, 72], and realistic rendering [130, 82, 104, 131, 129, 116].

Significant research on topics related to this proposal have been done under STC auspices, including development of the torn surface representation, research on surface-selfintersection, modeling operators using offsets and sweeps, and shape analysis. In addition to the above research results (C) has sponsored, in part, the Utah High School Computing Institute. a summer program designed to interest and challenge outstanding Utah high school sophomores and juniors wishing to explore areas of interest in computer science.

The research carried out under grants (D), (E), and (F) was partially directed toward mathematical problems motivated by questions in computer imaging. Under grant (F) work on the generic properties of solutions to the heat equation with applications to Gaussian blurring was conducted. Relevant papers from these grants are [30, 29, 33, 32, 35, 37, 41, 27, 28, 26, 56, 105, 88, 11, 18, 126, 54]. The application of methods of singularity theory have led to a fruitful interaction between the Departments of Computer Science and Mathematics at UNC. There have been several valuable aspects to this interaction including: a common seminar on geometry in computer imaging in which both the mathematics group and the computer science group give lectures.

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