CS 6320 Computer Vision Practice Set

Spring 2019
Name:
UID:

Each question carries 20 points. Show the necessary steps to compute the final solutions. You are free to use extra papers.

1. **Pose Estimation:** Let us consider a calibrated camera. Let the origin of the camera be given by O(0, 0, 0). The image resolution is 640 × 480 and the principal point is given by (320, 240). We assume the following parameters for the camera:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} 200 & 0 & 320 & 0 \\ 0 & 200 & 240 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathsf{I} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{pmatrix} \begin{pmatrix} X^m \\ Y^m \\ Z^m \\ 1 \end{pmatrix}$$

where (u, v) correspond to pixel coordinates and I denotes the 3 × 3 identity matrix.

$$\mathsf{K}^{-1} = \frac{1}{200} \begin{pmatrix} 1 & 0 & -320 \\ 0 & 1 & -240 \\ 0 & 0 & 200 \end{pmatrix}$$

The projections of two 3D points A and B on the image are given by $\mathbf{a}(120, 240)$ and $\mathbf{b}(320, 240)$, respectively. Find the coordinates of the two 3D points $A(X_1, Y_1, Z_1)$ and $B(X_2, Y_2, Z_2)$ that satisfy the 2 conditions: (1) the length of the line segment OA is given by $|OA| = 200$, and (2) the line segments OA and AB are perpendicular to each other. [20 points]

$$A = \lambda_1 K^{-1} \begin{bmatrix} a \\ 1 \end{bmatrix} = \lambda_1 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \qquad B = \lambda_2 K^{-1} \begin{bmatrix} b \\ 1 \end{bmatrix} = \lambda_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$|OA| = \sqrt{(-\lambda_1 - 0)^2 + 0 + (\lambda_1 - 0)^2} = 200$$

$$\Rightarrow \quad 2\lambda^2 = 40000 \Rightarrow \lambda = \pm 100\sqrt{2}$$

Since A is in front of camera its z value has to be positive $\Rightarrow$ $A = \begin{pmatrix} -100\sqrt{2} \\ 0 \\ 100\sqrt{2} \end{pmatrix}$

$$\overrightarrow{OA} \cdot \overrightarrow{AB} = 0 \quad \begin{bmatrix} -\lambda_1 \\ 0 \\ \lambda_1 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ 0 \\ \lambda_2 - \lambda_1 \end{bmatrix} = 0$$

$$-\lambda_1^2 + \lambda_1(\lambda_2 - \lambda_1) = 0$$

$$\lambda_2 = 2\lambda_1$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 200\sqrt{2} \end{pmatrix}.$$

2. **Model fitting:** In Figure **??**, you are given a set of 6 2D points (A, B, C, D, E, F). We can obtain a line equation by choosing 2 points at a time. Given two 2D points (x_1, y_1) and (x_2, y_2), we can obtain the line equation using the expression $(y - y_1)(x_1 - x_2) = (x - x_1)(y_1 - y_2)$. The distance of a point (x_0, y_0) from a line $ax + by + c = 0$ is given by $\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$. Let the threshold in selecting the inliers for a given line equation is given by **0.6**.

   - (a) Find a pair of points that gives the line equation with the maximum number of inliers. Show the line equations and inliers.
   - (b) Find a point pair that gives the line equation with the minimum number of inliers. Show the line equations and inliers.

   [20 points]

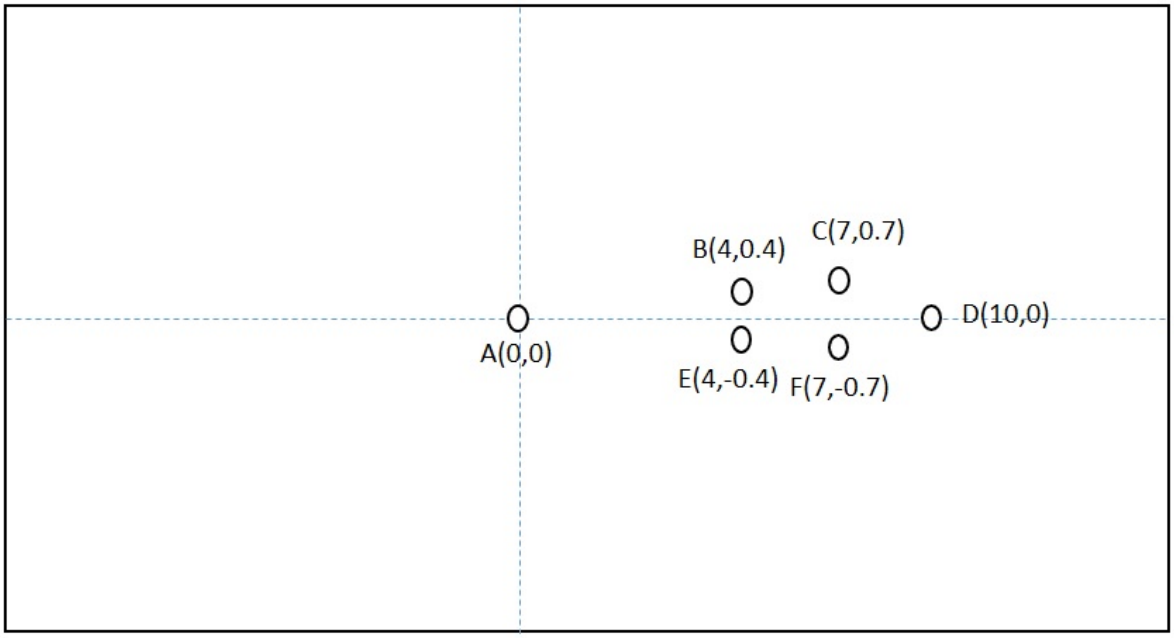


Figure 1:

Best line AD $\underline{y=0}$ Inliers = $\{A, B, E, D\}$

Worst line BE $\underline{x=4}$ Inliers = $\underline{\{B, E\}}$

3. **3D Modeling:** Let us consider a scenario where a 3D point **Q** is observed by two cameras. Let the 2 camera matrices be given by:

$$K_1 = K_2 = \begin{pmatrix} 200 & 0 & 320 \\ 0 & 200 & 240 \\ 0 & 0 & 1 \end{pmatrix}$$

Rotation matrices: $R_1 = R_2 = I$.

Translation matrices: $\mathbf{t}_1 = \mathbf{0}, \mathbf{t}_2 = (100, 0, 0)^T$.

The corresponding 2D points on the images are given by:

$$\mathbf{q_1} = \begin{pmatrix} 520 \\ 440 \\ 1 \end{pmatrix} \mathbf{q_2} = \begin{pmatrix} 320 \\ 440 \\ 1 \end{pmatrix}$$

$$K_1^{-1} = K_2^{-1} = \underline{\frac{1}{200}\begin{bmatrix} 1 & 0 & -320 \\ 0 & 1 & -240 \\ 0 & 0 & 200 \end{bmatrix}}$$

Compute the 3D point **Q**. [20 points]

$$Q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \lambda_1 K_1^{-1} [q_1]$$

$$= \lambda_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_1 \\ \lambda_1 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} + \lambda_2 K_2^{-1} [q_2]$$

$$Q_2 = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 100 \\ \lambda_2 \\ \lambda_2 \end{pmatrix}$$

$\underline{\text{Distsqr}} =$

$$\text{Dist}(Q_1, Q_2)^2 = (\lambda_1 - 100)^2 + 2(\lambda_1 - \lambda_2)^2$$

$$\frac{\partial \text{Distsqr}}{\partial \lambda_1} = 2(\lambda_1 - 100) + 4(\lambda_1 - \lambda_2) = 0$$

$$\Rightarrow \lambda_1 - 100 + 2\lambda_1 - 2\lambda_2 = 0$$

$$\underline{3\lambda_1 - 2\lambda_2 - 100 = 0}$$

$$\frac{\underline{\partial \text{Distsqr}}}{\partial \lambda_2} = 4(\lambda_1 - \lambda_2)(-1) = 0$$

$$\underline{\lambda_1 = \lambda_2}$$

$$\lambda_1 = 100$$

$$\underline{\underline{\lambda_2 = 100}}$$

$$Q = \left(\frac{Q_1 + Q_2}{2}\right) = \begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix}$$

4. **Vocabulary tree:** We are given three images I_1, I_2 and I_3. Each of these images have two 2D descriptors as given below.

$$I1 : \{(1.5, 0.5), (-1.5, -1.5)\}, \quad I2 : \{(1.5, 1.5), (1.5, -1.5)\}, \quad I3 : \{(1.5, 0.5), (1.5, -0.5)\}$$

In Figure ??, we show a vocabulary tree with branch factor 4. Using this vocabulary tree find the best image match among the 3 possible pairs $\{(I_1, I_2), (I_1, I_3), (I_2, I_3)\}$. Assume that the nodes in the tree has the same weight $w_i = 1$. Show the normalized difference in each of the three pairs. [20 points]

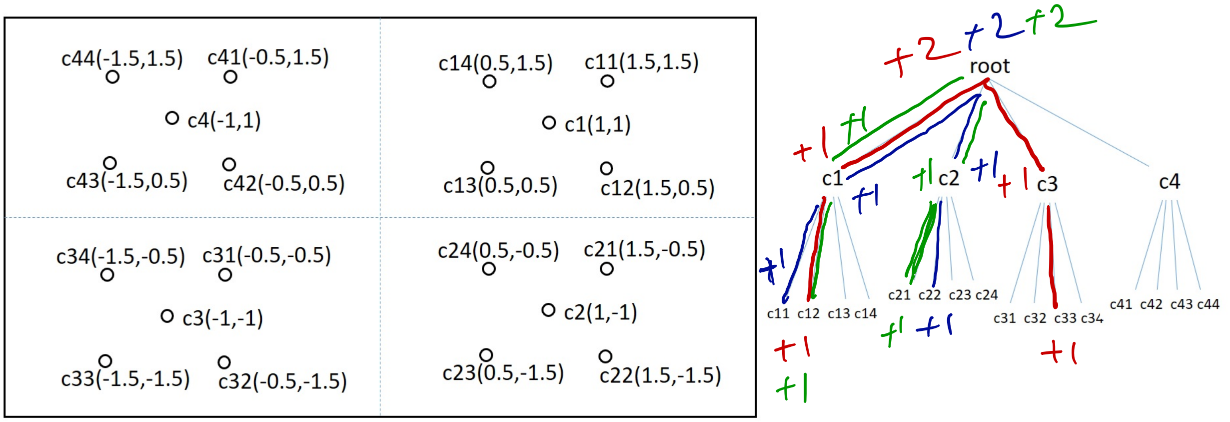


Figure 2:

$q_1 = [2, 1, 0, 1, 0, 0, 1\ 0\ 0 \quad 0\ 0\ 0\ 0 \quad 0\ 0\ 1\ 0 \quad 0\ 0\ 0\ 0]$

$q_2 = [2, 1, 1, 0, 0 \quad 1, 0, 0, 0 \quad 0\ 1, 0\ 0 \quad 0\ 0\ 0\ 0 \quad 0\ 0\ 0\ 0]$

$q_3 = [2, 1, 1, 0, 0, \quad 0\ 1\ 0\ 0 \quad 1\ 0\ 0\ 0 \quad 0\ 0\ 0\ 0 \quad 0\ 0\ 0\ 0]$

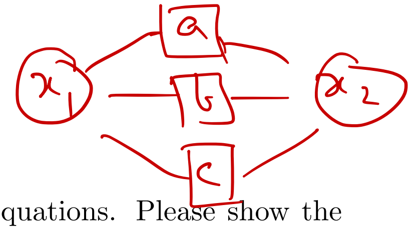
$$\text{Norm-Diff}(q_1, q_2) = \frac{q_1}{2} - \frac{q_2}{2} = \frac{6}{2} = 3$$

$$\text{Norm-Diff}(q_1, q_3) = \frac{q_1}{2} - \frac{q_2}{2} = \frac{4}{2} = 2$$

$$\text{Norm-Diff}(q_2, q_3) = q_2/2 - q_3/2 = 4/2 = \underline{\underline{2}}$$

5. **Belief Propagation:** You have 2 Boolean variables ($x_1, x_2 \in \{0, 1\}$) and 3 equations as shown below:

$$\begin{aligned} x_1 + 4x_2 &= 5 \\ -x_1 + 2x_2 &= 1 \\ -3x_1 + 2x_2 &= -1.2 \end{aligned}$$



Show the factor graph and use Belief propagation to solve the equations. Please show the messages in each iteration till the algorithm terminates. [20 points]

$$\begin{array}{c|cc} & x_2 & \\ \hline x_1 & 5 & 1 \\ & 4 & 0 \end{array} \qquad \begin{array}{c|cc} & x_2 & \\ \hline x_1 & 1 & 1 \\ & 2 & 0 \end{array} \qquad \begin{array}{c|cc} & x_2 & \\ \hline x_1 & 1.2 & 3.2 \\ & 1.8 & 0.2 \end{array}$$

$\boxed{a}$ $\boxed{b}$ $\boxed{c}$

The tables are constructed using absolute values for the difference.

Step 1 All zero messages

$m_{1\to a} = 0$, $m_{2\to a} = 0$, $m_{1\to b} = 0$, $m_{2\to b} = 0$,

$m_{1\to c} = 0$, $m_{2\to c} = 0$

——— x ———

Step 2

$$m_{a\to 1} = \min_{x_2} \begin{bmatrix} m_{2\to a}(x_2) + C_a(0, x_2) \\ m_{2\to a}(x_2) + C_a(1, x_2) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$m_{a\to 2} = \min_{x_1} \begin{bmatrix} m_{1\to a}(x_1) + C_a(x_1, 0) \\ m_{1\to a}(x_1) + C_a(x_1, 1) \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$m_{b\to 1} = \min_{x_2} \begin{bmatrix} m_{2\to b}(x_2) + C_b(0, x_2) \\ m_{2\to b}(x_2) + C_b(1, x_2) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Similarly $m_{b\to 2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $m_{c\to 1} = \begin{pmatrix} 1.2 \\ 0.2 \end{pmatrix}$ $m_{c\to 2} = \begin{pmatrix} 1.2 \\ 0.2 \end{pmatrix}$

Step 3 $b_1 = \begin{pmatrix} 3.2 \\ 0.2 \end{pmatrix}$ $b_2 = \begin{pmatrix} 6.2 \\ 0.2 \end{pmatrix}$

Step 4

$x_1 \leftarrow 1$, $x_2 \leftarrow 1$

Step 5 :

$$m_{1\to a} = \begin{pmatrix} 2.2 \\ 0.2 \end{pmatrix} \quad m_{2\to a} = \begin{pmatrix} 2.2 \\ 0.2 \end{pmatrix}$$

$$m_{1\to b} = \begin{pmatrix} 2.2 \\ 0.2 \end{pmatrix} \quad m_{2\to b} = \begin{pmatrix} 5.2 \\ 0.2 \end{pmatrix}$$

$$m_{1\to c} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad m_{2\to c} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

Step 6 :

$$m_{a\to 1} = \min_{x_2} \begin{bmatrix} m_{2\to a}(x_2) + C_b(0, x_2) \\ m_{2\to a}(x_2) + C_b(1, x_2) \end{bmatrix}$$

$$= \begin{pmatrix} 1.2 \\ 0.2 \end{pmatrix}$$

$$m_{a\to 2} = \min_{x_1} \begin{bmatrix} m_{1\to a}(x_1) + C_b(x_1, 0) \\ m_{1\to a}(x_1) + C_b(x_1, 1) \end{bmatrix}$$

$$= \begin{pmatrix} 4.2 \\ 0.2 \end{pmatrix}$$

$$m_{b\to 1} = \min_{x_2} \begin{bmatrix} m_{2\to b}(x_2) + C_b(0, x_2) \\ m_{2\to b}(x_2) + C_b(1, x_2) \end{bmatrix}$$

$$= \begin{pmatrix} 1.2 \\ 0.2 \end{pmatrix}$$

$$m_{b\to 2} = \min_{x_1} \begin{bmatrix} m_{1\to b}(x_1) + C_b(x_1, 0) \\ m_{1\to b}(x_1) + C_b(x_1, 1) \end{bmatrix}$$

$$= \begin{pmatrix} 1.2 \\ 0.2 \end{pmatrix}$$

$$m_{c\to 1} = \begin{pmatrix} 3.2 \\ 0.2 \end{pmatrix} \quad m_{c\to 2} = \begin{pmatrix} 1.8 \\ 0.2 \end{pmatrix}$$

Step 7: $b_1 = \begin{pmatrix} 5.6 \\ 0.6 \end{pmatrix} \quad b_2 = \begin{pmatrix} 8.2 \\ 0.6 \end{pmatrix}$

Step 8 : $x_1 \leftarrow 1, \quad x_2 \leftarrow 1,$ **TERMINATE**