

# Stereo Matching

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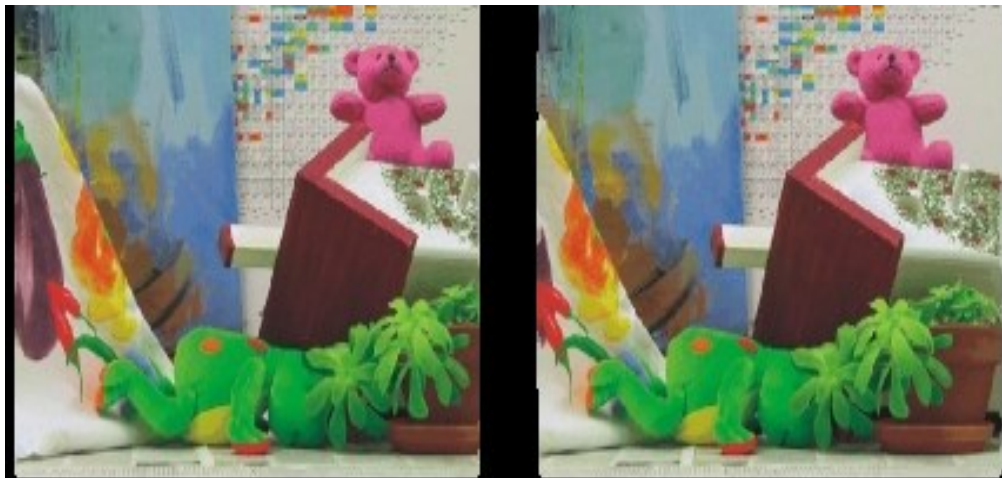
# Reference

- Daniel Scharstein and Richard Szeliski, A Taxonomy and Evaluation of Dense Two-Frame Stereo Correspondence Algorithms, IJCV, 2002.

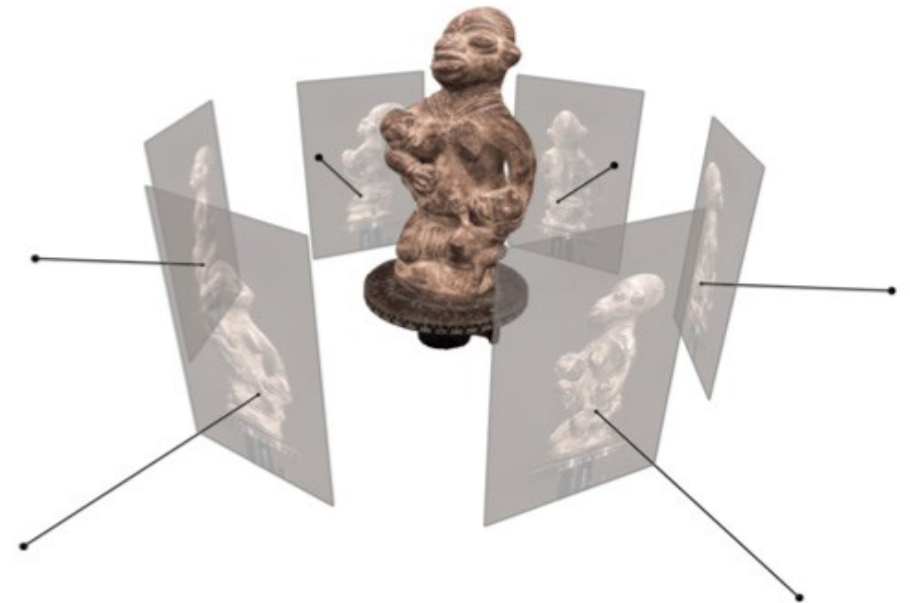
<http://vision.middlebury.edu/stereo/taxonomy-IJCV.pdf>

# Dense Correspondence in Computer Vision

## Binocular Stereo



## Multiview stereo



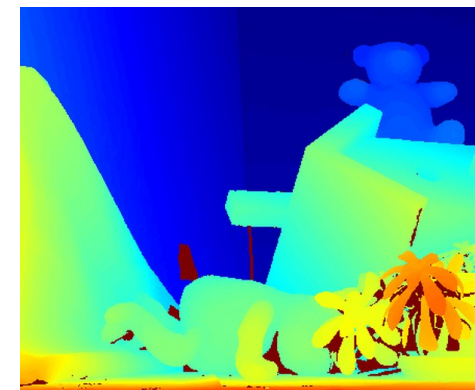
# Stereo Matching



Left (reference)



Right



Left Disparity Map

- Dense pixel correspondence in rectified pairs
- Disparity Map:  $D(x, y)$  (*treated as +ve*)

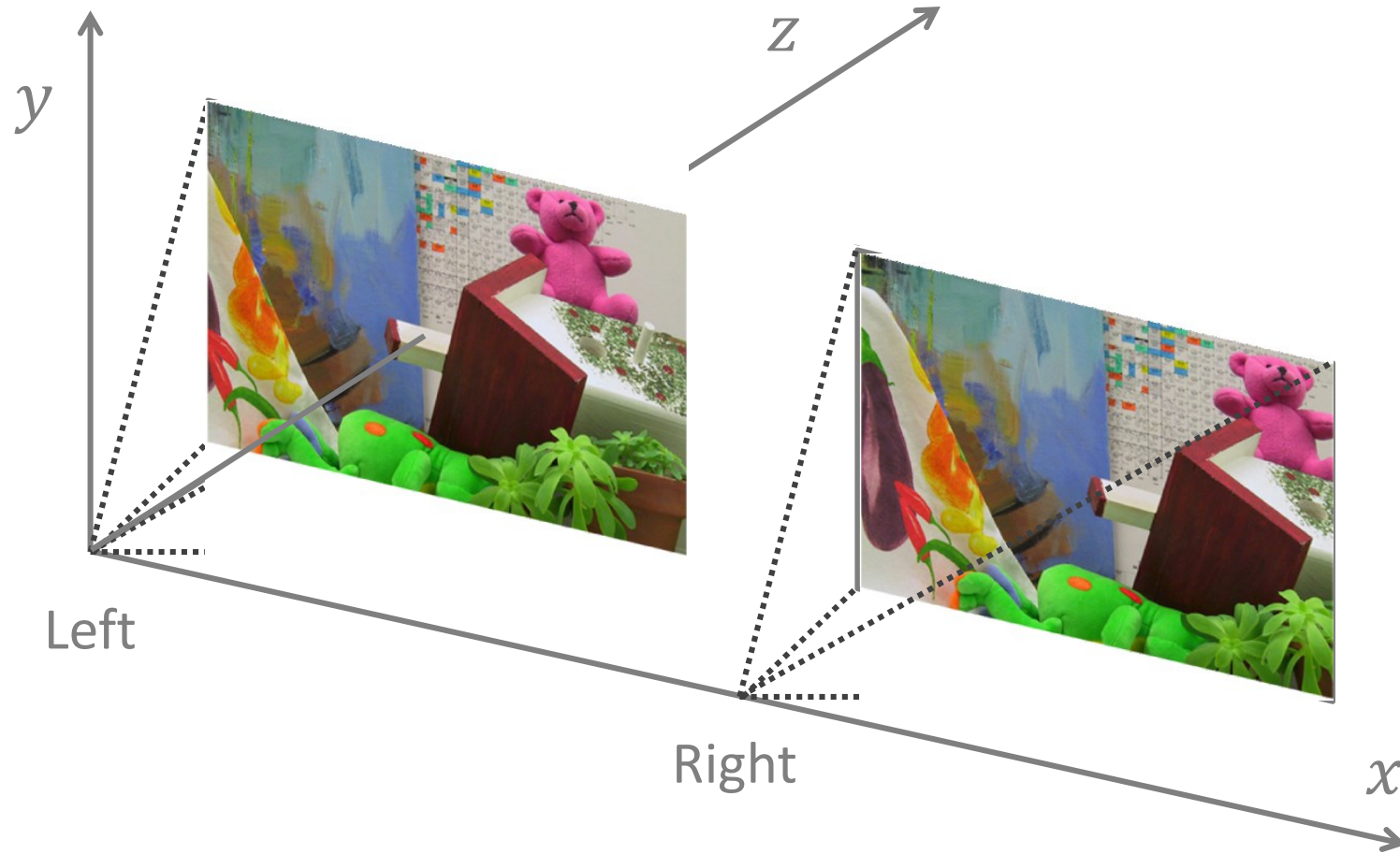
$$x' = x - D(x, y), \quad y' = y$$

- Depth Map:  $Z(x, y) = \frac{\text{baseline} * \text{focal length}}{D(x, y)}$

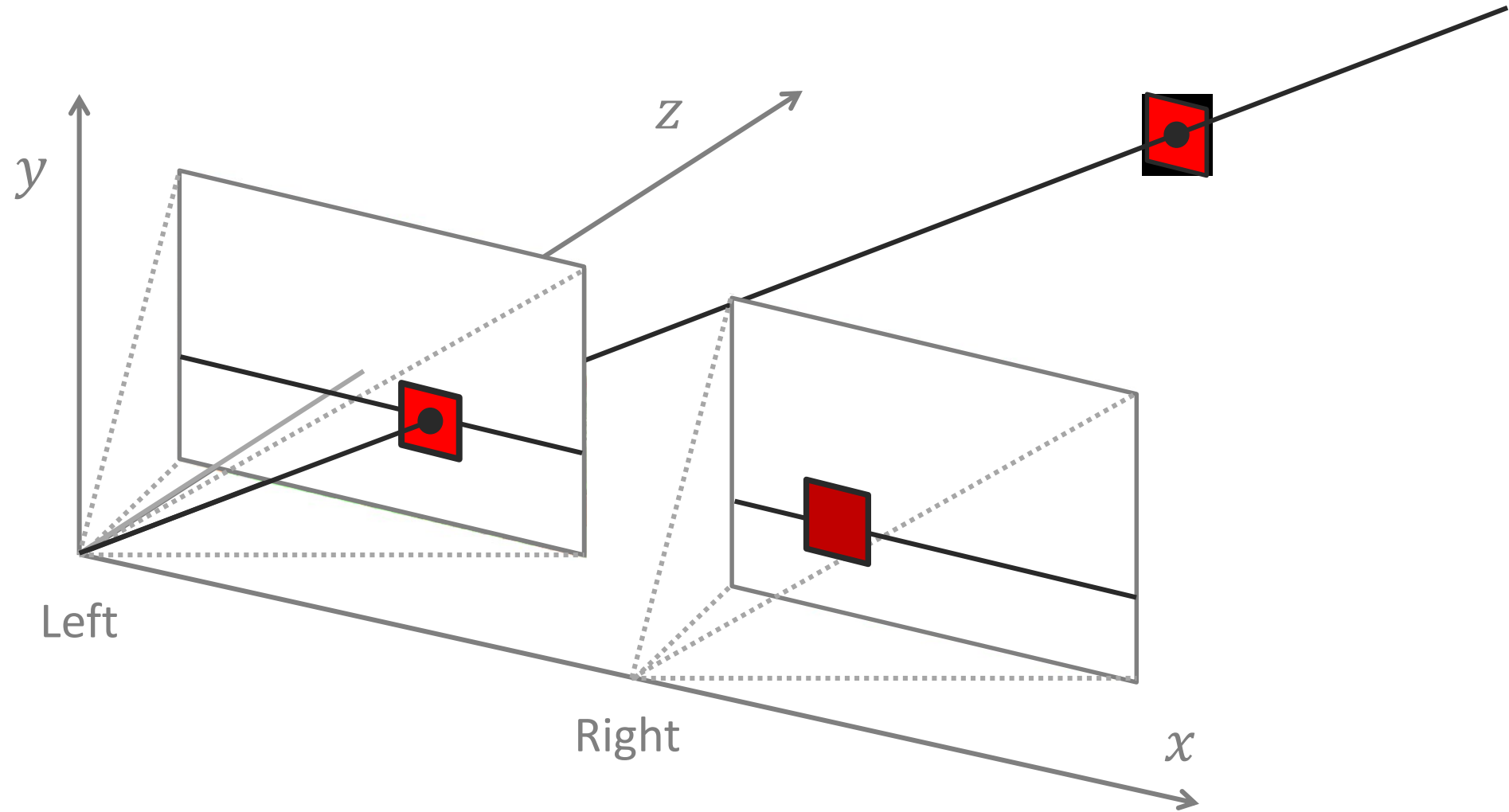


Depth Map

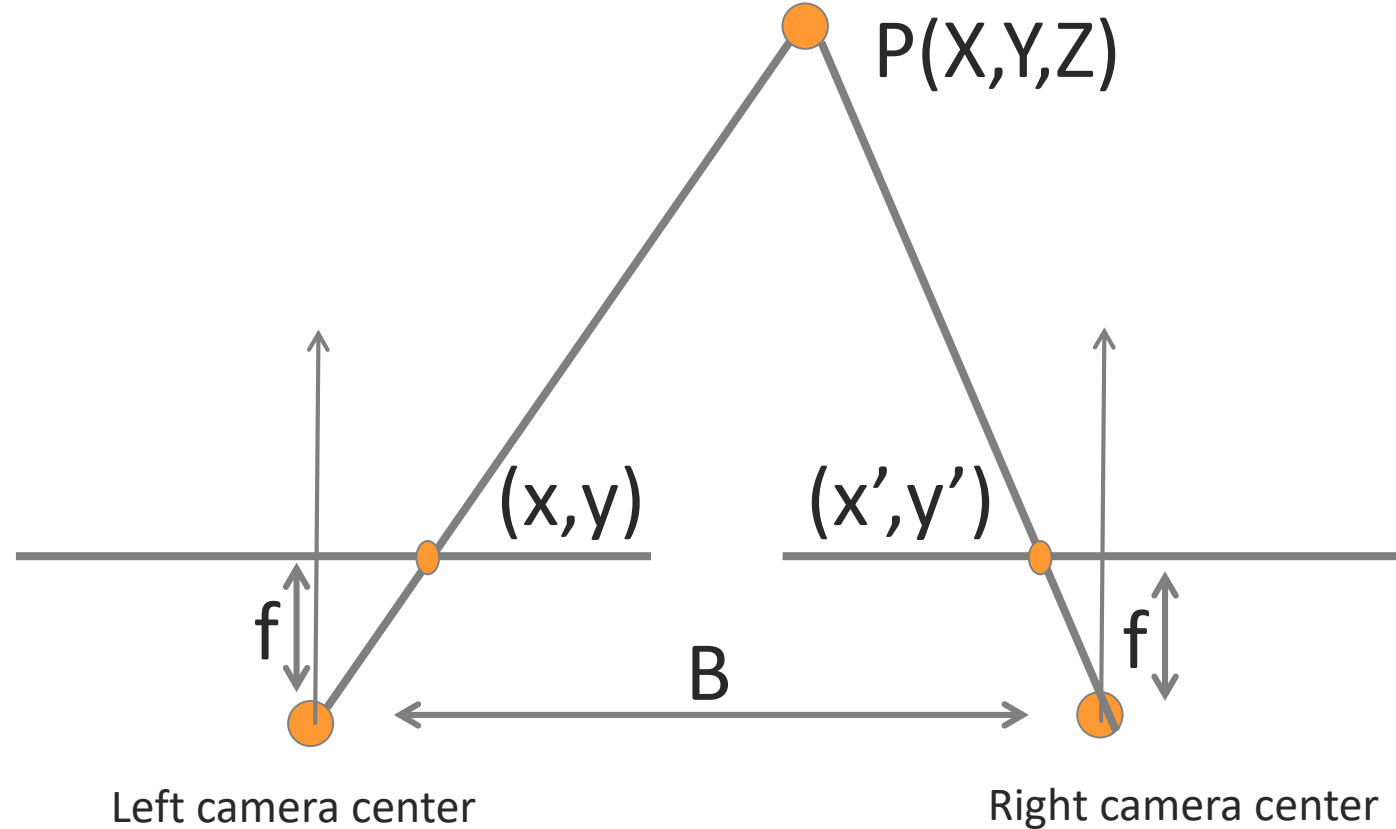
# Binocular Stereo Matching



# Binocular Stereo Matching



# Binocular stereo matching (Top View)



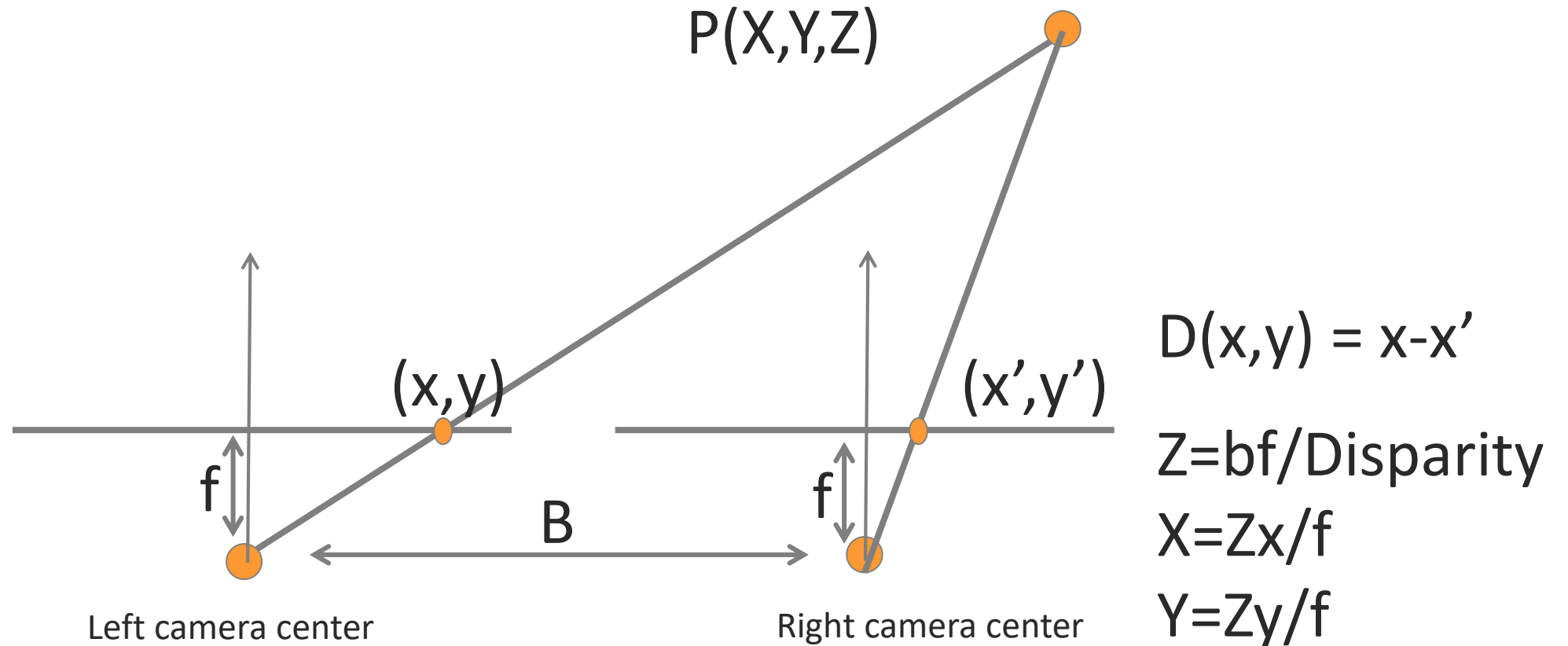
$$D(x, y) = x - x'$$

$$Z = bf / \text{Disparity}$$

$$X = Zx / f$$

$$Y = Zy / f$$

# Binocular stereo matching (Top View)





# Matching costs $C(x,y,d)$

- Find pairs of pixels (or local patches) with similar appearance
- Minimize *matching cost* (maximize photo-consistency)

- Patch-based** (parametric vs non-parametric)

- Sum of Absolute Difference (SAD),
- Sum of Squared Difference (SSD),
- Normalized Cross Correlation (ZNCC)

$$C_{SAD}(\mathbf{p}, \mathbf{d}) = \sum_{\mathbf{q} \in N_p} |I_L(\mathbf{q}) - I_R(\mathbf{q} - \mathbf{d})|$$

$$C_{ZNCC}(\mathbf{p}, \mathbf{d}) =$$

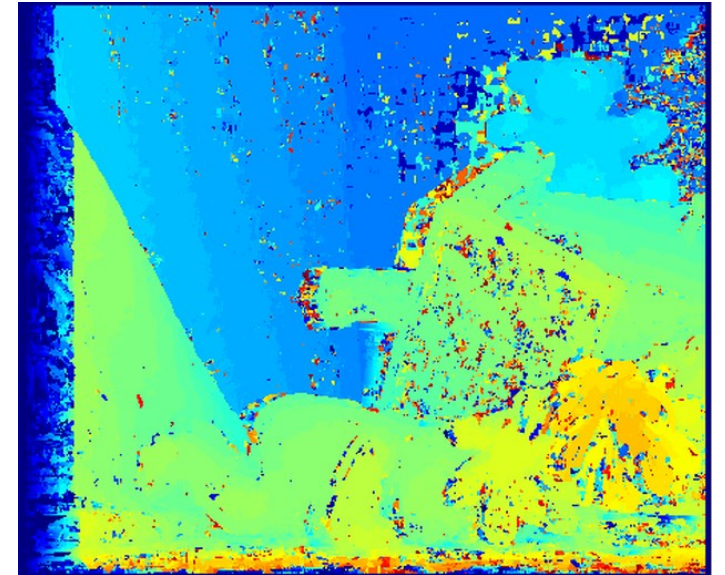
$$\frac{\sum_{\mathbf{q} \in N_p} (I_L(\mathbf{q}) - \bar{I}_L(\mathbf{p})) (I_R(\mathbf{q} - \mathbf{d}) - \bar{I}_R(\mathbf{p} - \mathbf{d}))}{\sqrt{\sum_{\mathbf{q} \in N_p} (I_L(\mathbf{q}) - \bar{I}_L(\mathbf{p}))^2 \sum_{\mathbf{q} \in N_p} (I_R(\mathbf{q} - \mathbf{d}) - \bar{I}_R(\mathbf{p} - \mathbf{d}))^2}}$$

- Descriptor-based**

- (hand-crafted features) SIFT, DAISY, ..
- (learnt features) Deep learning (revisit later)

# Local Optimization

- Minimize matching cost at each pixel in the left image independently
- Winner-take-all (WTA)



# Local evidence not enough ...

- Photometric Variations →
- Reflections
- Transparent surfaces
- Texture-less Areas
- Non-Lambertian Surfaces
- Repetitive patterns
- Complex Occlusions



(Image Source: Lectures on stereo matching, Christian Unger and Nassir Navab, TU Munchen)  
[http://campar.in.tum.de/twiki/pub/Chair/TeachingWs09Cv2/3D\\_CV2\\_WS\\_2009\\_Stereo.pdf](http://campar.in.tum.de/twiki/pub/Chair/TeachingWs09Cv2/3D_CV2_WS_2009_Stereo.pdf)

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# Global Optimization

- Solve for all disparities simultaneously ...
- Solve a pixel labeling problem
- Labels are discrete (ordered),  $d \in L_D$

$$L_D = [d_{min}, d_{max}]$$

- Incorporate regularization into objective

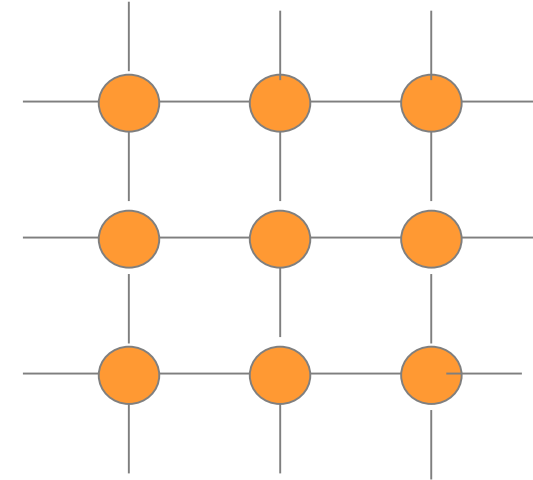
$$E(d) = E_{data}(d) + \lambda E_{smooth}(d)$$

- Data term encodes matching costs
- Smoothness term encodes priors
  - Encourage adjacent pixels to take similar disparities

# Disparity Computation

Let the disparity map be given by  $d$ , where  $d(x,y)$  denotes the disparity for a pixel in the left image at location  $(x,y)$ .

Let all the edges in the image graph be given by  $E$  by treating the image as a 4-connected or 8-connected graph:



$$E(d) = E_{data}(d) + \lambda E_{smooth}(d)$$

Data term:

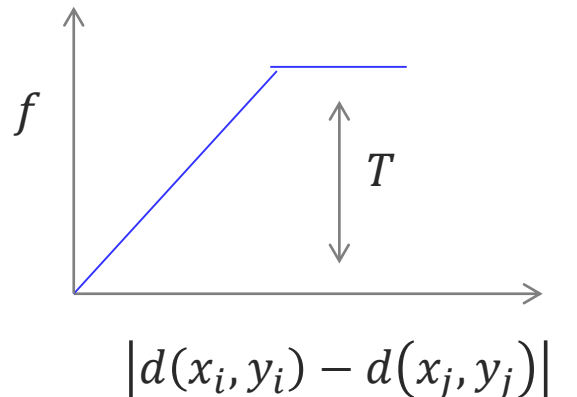
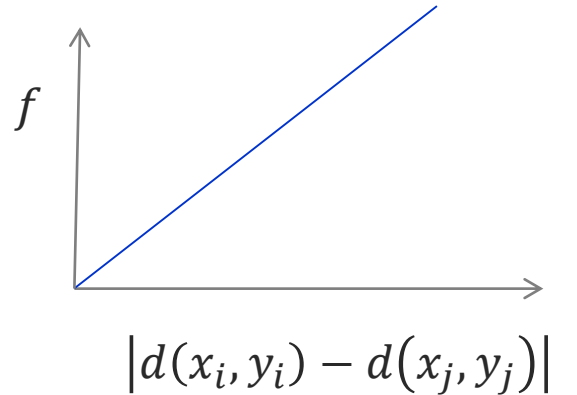
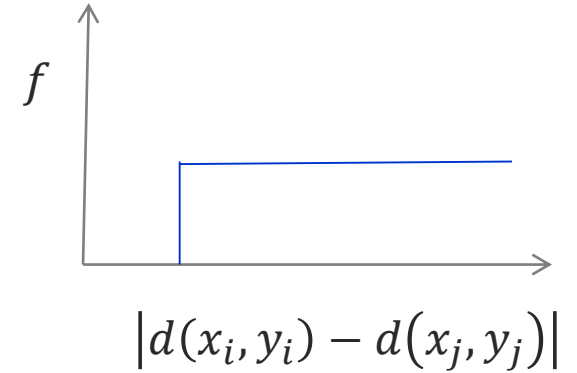
$\lambda$  is the constant term that can be manually fixed or learned from data.

$$E_{data}(d) = \sum_{(x,y)} C(x, y, d(x, y))$$

# Different smoothness terms

$$E_{smooth}(d) = \sum_{(i,j) \in E} f(d(x_i, y_i), d(x_j, y_j))$$

$$diff = |d(x_i, y_i) - d(x_j, y_j)|$$



Potts:  $f = 1$  if  $diff > 0$  else  $f = 0$  (Robust)

Linear:  $f = diff$  (Not Robust)

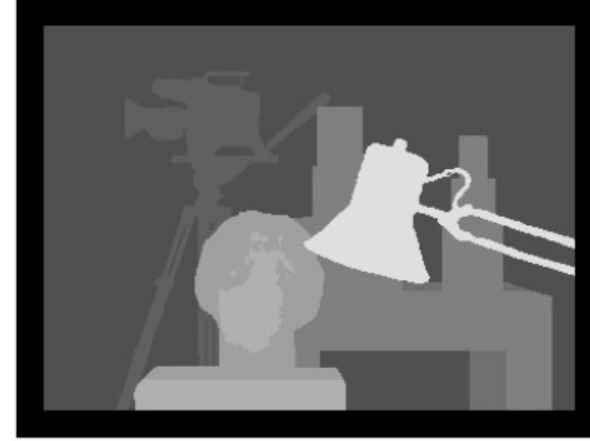
Truncated model:  $f = diff$  if  $diff < T$  else  $f = T$   
(Robust)



# Stereo Image with Groundtruth



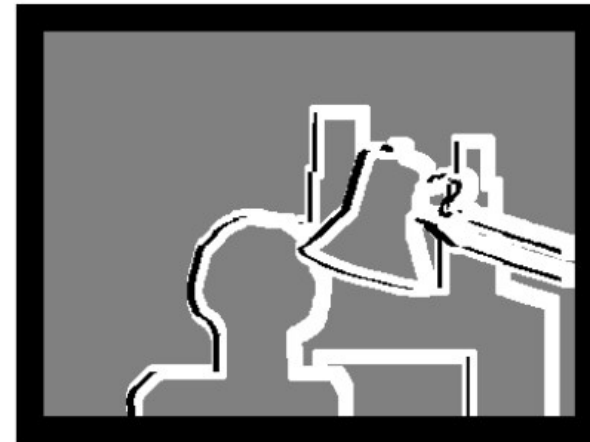
Left image from orig. stereo pair



Groundtruth disparity



Texture-less regions



Depth discontinuities (white) and  
Occlusion (Black)

# BP Results



True disparities



Belief propagation

# Sample Problem (not the same as real setup)

You are given two images: I1 (2x2 pixel grid) and I2 (2x3 pixel grid) as shown in Figure 1. Find the match for every pixel in the first image I1. Every pixel  $p(x, y)$  in I1 can be matched to a pixel  $p'(x, y)$  or  $p'(x + 1, y)$  in I2. In other words, every pixel in I1 can have only two disparity states  $[0, 1]$ : 0 when  $p(x, y)$  is matched to  $p'(x, y)$ , and 1 when  $p(x, y)$  is matched to  $p'(x + 1, y)$ . The unary for a pixel  $p(x, y)$  (cost function that depends only on a single pixel in I1) is given by:

$$U(0) = |p(x, y) - p'(x, y)|, \quad U(1) = |p(x, y) - p'(x + 1, y)|$$

The pairwise function depends on the states of two nearby pixels (in the image I1) and is given by:

$$P(0, 0) = 0, \quad P(0, 1) = 10, \quad P(1, 0) = 10, \quad P(1, 1) = 0$$

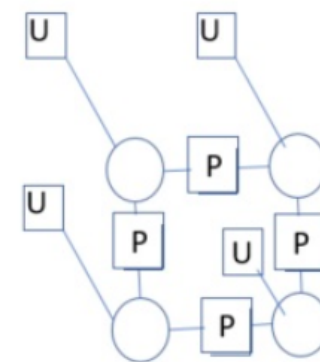
Use Belief propagation to solve the matching problem. Please show the messages in each iteration till the algorithm terminates.

50	0
50	0

I1

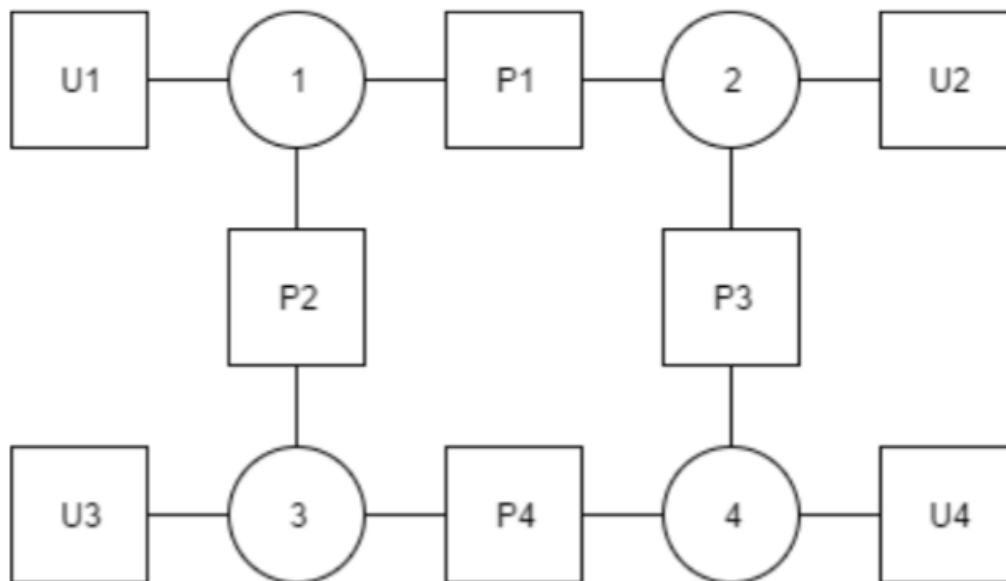
50	0	0
0	50	0

I2



Factor graph

# Solution



u1	0	p1	0	10
	50		10	0
u2	0	p2	0	10
	0		10	0
u3	50	p3	0	10
	0		10	0
u4	50	p4	0	10
	0		10	0

# Solution

## First Iteration

$m_{1 \rightarrow u1}$	0	$m_{1 \rightarrow p1}$	0	$m_{1 \rightarrow p2}$	0		
	0		0		0		
$m_{2 \rightarrow u2}$	0	$m_{2 \rightarrow p1}$	0	$m_{2 \rightarrow p3}$	0		
	0		0		0		
$m_{3 \rightarrow u3}$	0	$m_{3 \rightarrow p2}$	0	$m_{3 \rightarrow p4}$	0		
	0		0		0		
$m_{4 \rightarrow u4}$	0	$m_{4 \rightarrow p3}$	0	$m_{4 \rightarrow p4}$	0		
	0		0		0		
$m_{u1 \rightarrow 1}$	0	$m_{p1 \rightarrow 1}$	0	$m_{p2 \rightarrow 1}$	0		
	50		0		0		
$m_{u2 \rightarrow 2}$	0	$m_{p1 \rightarrow 2}$	0	$m_{p3 \rightarrow 2}$	0		
	0		0		0		
$m_{u3 \rightarrow 3}$	50	$m_{p2 \rightarrow 3}$	0	$m_{p4 \rightarrow 3}$	0		
	0		0		0		
$m_{u4 \rightarrow 4}$	50	$m_{p3 \rightarrow 4}$	0	$m_{p4 \rightarrow 4}$	0		
	0		0		0		
$b_1$	0	$b_2$	0	$b_3$	50	$b_4$	50
	50		0		0		0
$x_1$	0	$x_2$	0	$x_3$	1	$x_4$	1

## Second Iteration:

$m_{1 \rightarrow u1}$	0	$m_{1 \rightarrow p1}$	0	$m_{1 \rightarrow p2}$	0		
	0		50		50		
$m_{2 \rightarrow u2}$	0	$m_{2 \rightarrow p1}$	0	$m_{2 \rightarrow p3}$	0		
	0		0		0		
$m_{3 \rightarrow u3}$	0	$m_{3 \rightarrow p2}$	50	$m_{3 \rightarrow p4}$	50		
	0		0		0		
$m_{4 \rightarrow u4}$	0	$m_{4 \rightarrow p3}$	50	$m_{4 \rightarrow p4}$	50		
	0		0		0		
$m_{u1 \rightarrow 1}$	0	$m_{p1 \rightarrow 1}$	0	$m_{p2 \rightarrow 1}$	10		
	50		0		0		
$m_{u2 \rightarrow 2}$	0	$m_{p1 \rightarrow 2}$	0	$m_{p3 \rightarrow 2}$	10		
	0		10		0		
$m_{u3 \rightarrow 3}$	50	$m_{p2 \rightarrow 3}$	0	$m_{p4 \rightarrow 3}$	10		
	0		10		0		
$m_{u4 \rightarrow 4}$	50	$m_{p3 \rightarrow 4}$	0	$m_{p4 \rightarrow 4}$	10		
	0		0		0		
$b_1$	10	$b_2$	10	$b_3$	60	$b_4$	60
	50		10		10		0
$x_1$	0	$x_2$	0	$x_3$	1	$x_4$	1