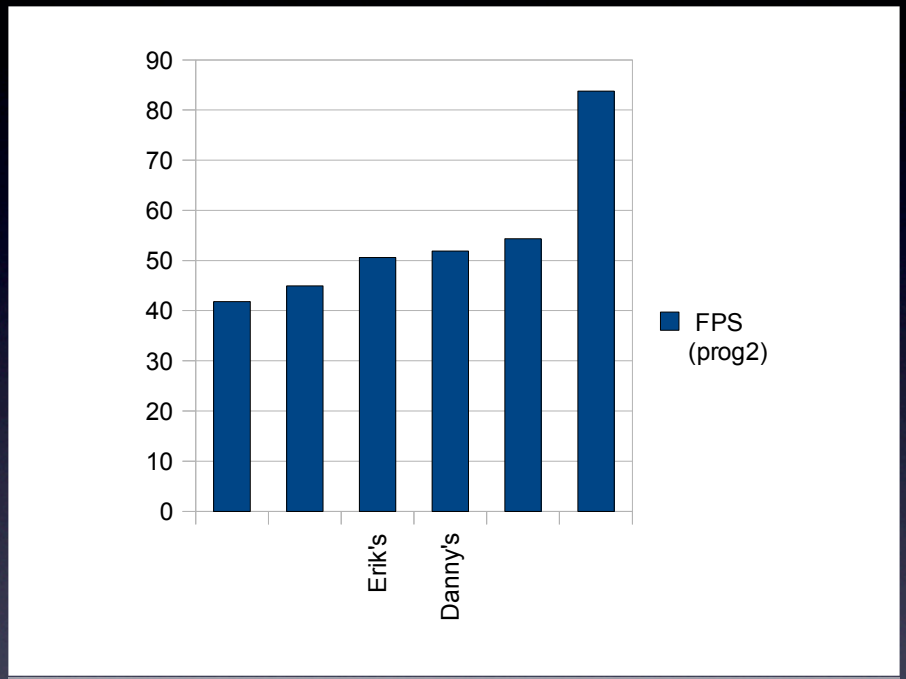
Texturing

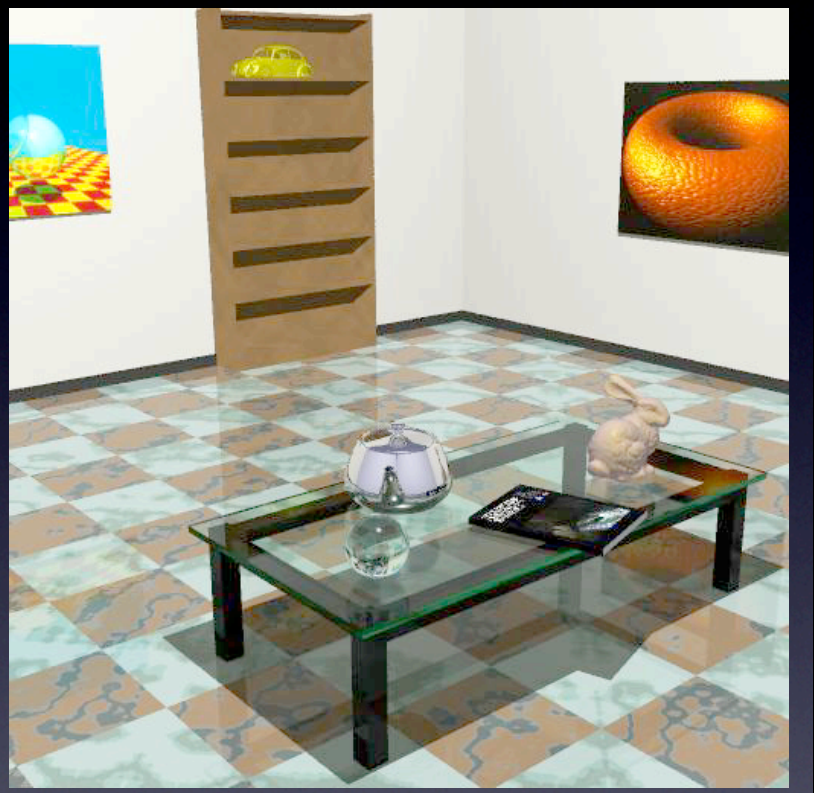
CS 6965 Fall 2011



Program 2

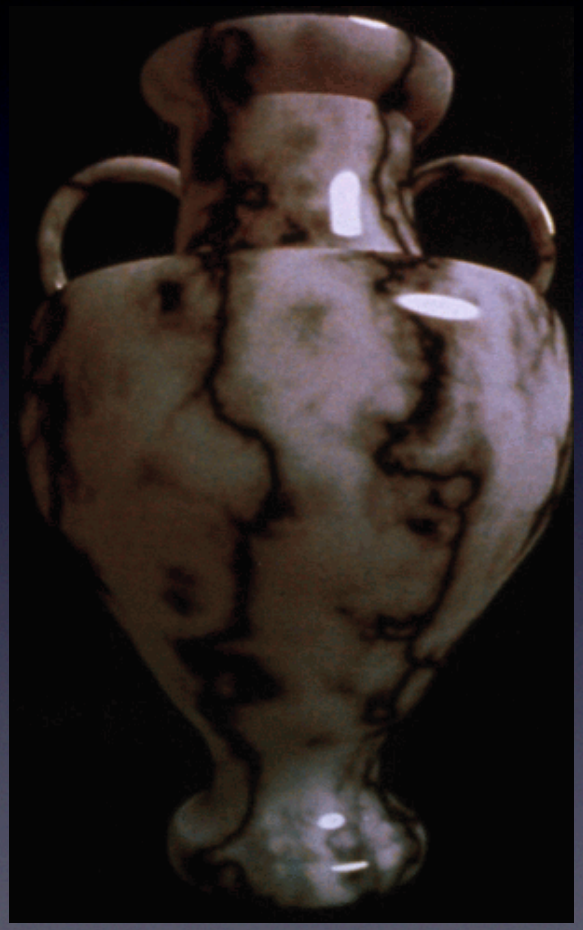
Texture mapping

- Most real objects do not have uniform color
- Texture: color and other material properties as a function of space



Texture mapping topics

- Image textures
- Texture coordinates
  - Linear, cylindrical, spherical mappings
  - Barycentric coordinates for triangles
  - Software architecture
- Procedural textures
  - Simple: checkerboards, tiles, etc.
  - Fractal noise based: marble, granite, wood, etc.
- Bump mapping



Ken Perlin, NYU

Lambertian Shading

Compute hit position ($\vec{P} = \vec{O} + t\vec{V}$)

Call primitive to get normal ($\vec{\mathrm{N}}$) (normalized)

$costheta = -\vec{N} \cdot \vec{V}$

$if(costheta < 0)$

    normal =-normal

Color light = scene.ambient*Ka

foreach light source

    get $\mathrm{C_L}$ and $\vec{\mathrm{L}}$

    dist=$\|\vec{\mathrm{L}}\|, \overrightarrow{L_n} = \dfrac{\vec{L}}{\|\vec{L}\|}$

    $cosphi = \vec{N} \cdot \overrightarrow{L_n}$

    if($cosphi > 0$)

        if(!intersect with $0 < t < dist$)

            light += $\mathrm{C_L} * (Kd * cosphi)$

result=light*surface color

Simple image texture

Use hit position to determine image location

$\vec{P}$: hit position

$\vec{C}$: lower left corner of image

$\vec{U}$: image X axis in world space

$\vec{V}$: image Y axis in world space

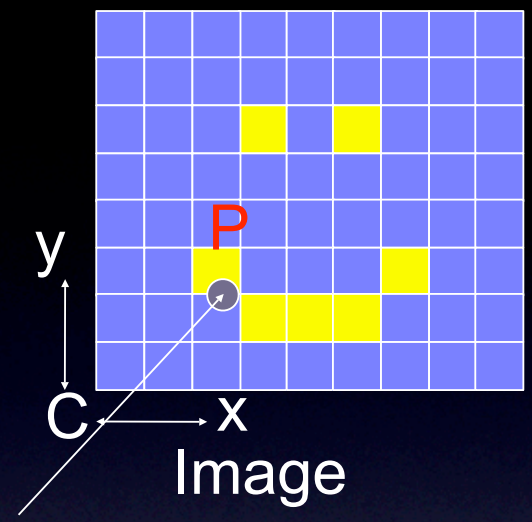
x, y: normalized image coordinates (0-1)

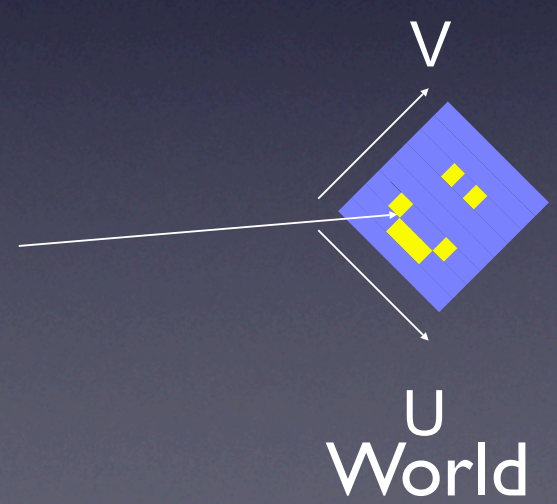
$$\vec{P} = \vec{C} + x\vec{U} + y\vec{V}$$

if $\vec{U} \cdot \vec{V} = 0$:

$$x = \frac{\left(\vec{P} - \vec{C}\right) \cdot \vec{U}}{\left\|\vec{U}\right\|}$$

$$y = \frac{\left(\vec{P} - \vec{C}\right) \cdot \vec{V}}{\left\|\vec{V}\right\|}$$

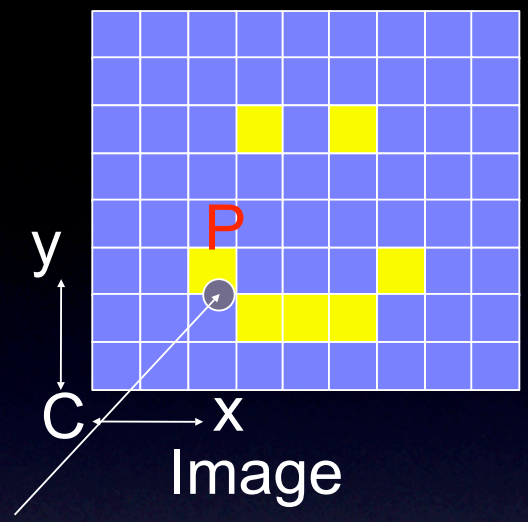




Simple image texture

$$x = \frac{\left(\vec{P} - \vec{C}\right) \cdot \vec{U}}{\left\|\vec{U}\right\|}$$

$$y = \frac{\left(\vec{P} - \vec{C}\right) \cdot \vec{V}}{\left\|\vec{V}\right\|}$$



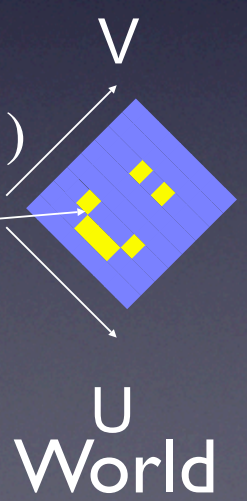
$$ix = (\text{int})\left(x\left(xres - 1\right)\right), fx = ix - x\left(xres - 1\right)$$

$$iy = (\text{int})\left(y\left(yres - 1\right)\right), fy = iy - y\left(yres - 1\right)$$

Bilinear interpolation of image(ix,iy) - image(ix+1,iy+1)

using fx,fy as interpolation factors

Use interpolated color in lambertian shading



Miscellaneous details

- What happens for points outside of the texture?
  - Tile texture (mod function)
  - Clamp (stretch out last value)
  - Use an "outside" color
- What if U/V are not orthogonal?
  - Solve a 2x2 matrix to get x/y
- Other interpolations:
  - Nearest neighbor
  - Higher order interpolation
  - Filter reconstruction (what filter width?)

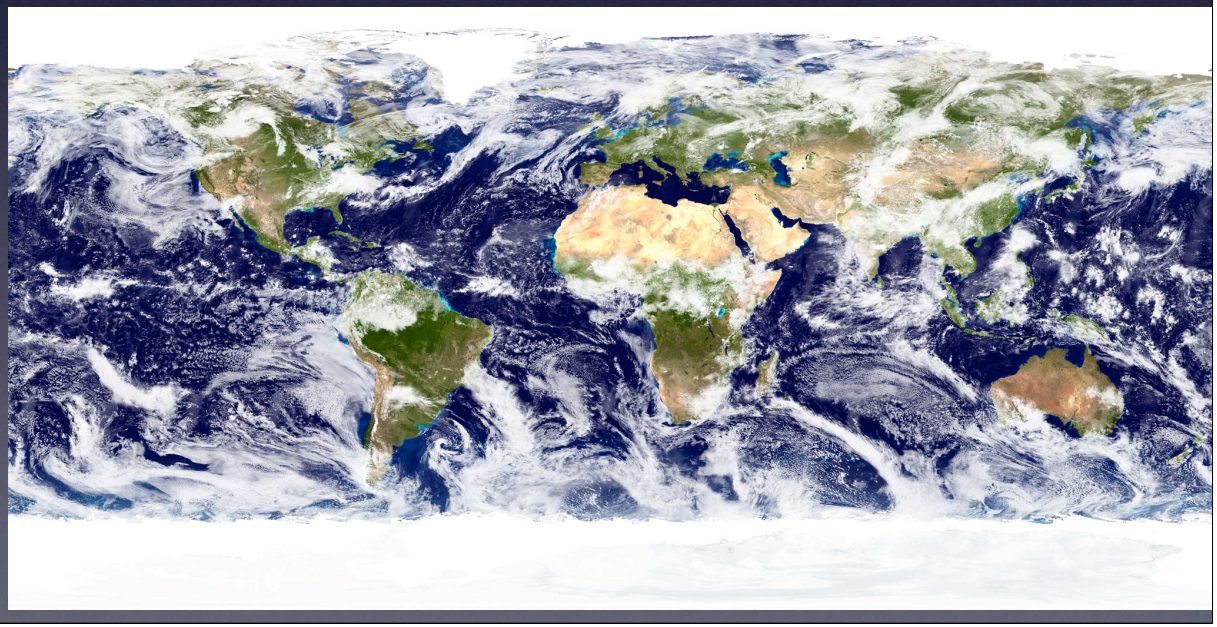
Texturing process

- This example illustrated two orthogonal issues of texturing for ray tracing:
  - Mapping hit position to image space (coordinate transformation), example: linear
  - Mapping texture coordinate to color or other attributes, example: linear interpolation of an image

Texture coordinates

- Textures are not always flat
- Textures live in a space called "uvw"
- Texture coordinate mapping: xyz to uvw
- Texture space may not be linear

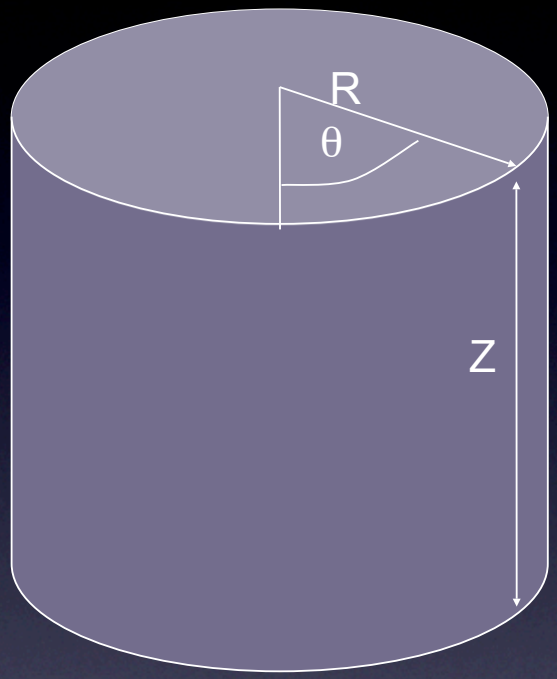
NASA Blue marble project



Cylinder projection

$$x = \cos(\theta)$$

$$y = \sin(\theta)$$



inverse mapping:

$$R = \sqrt{x^2 + y^2}$$

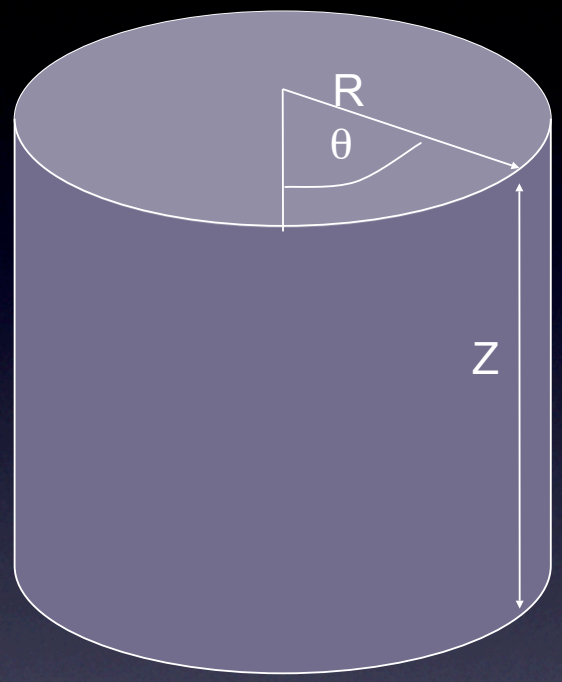
$$\theta = \operatorname{atan}2(y, x)$$

$\operatorname{atan}2$: robust atan(y/x) for all quadrants

Cylinder projection

$x = \cos\theta$

$y = \sin\theta$



inverse mapping:

$R = \sqrt{x^2 + y^2}$

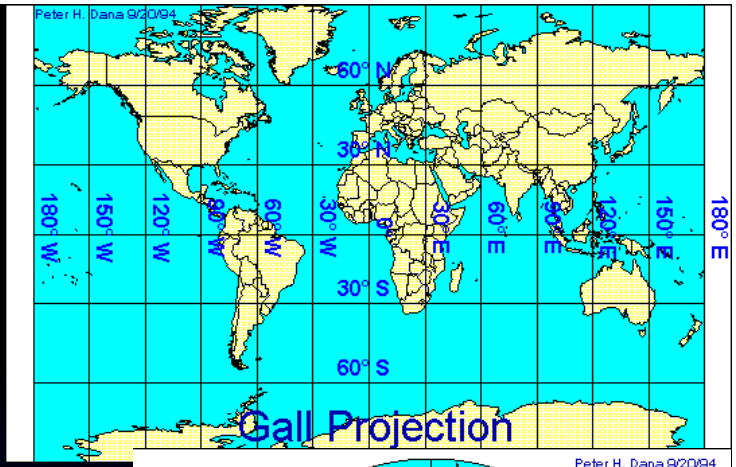
$\theta = \operatorname{atan}2(y,x)$

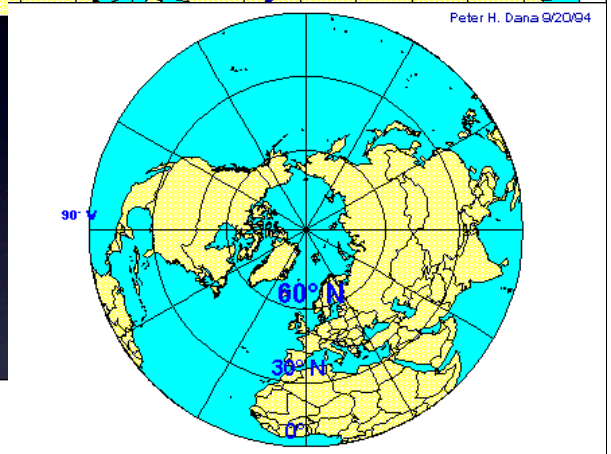
$\operatorname{atan}2$: robust atan(y/x) for all quadrants

$u = \dfrac{\theta}{2\pi}$;if(u<1)u+=1, $v = z, w = R$

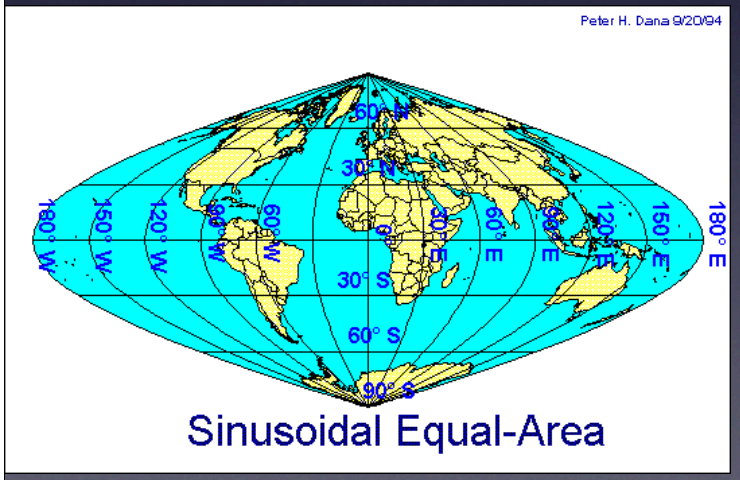
Sphere mapping

- Lots of ways to map a plane (image) to a sphere
- Remember grade school geography

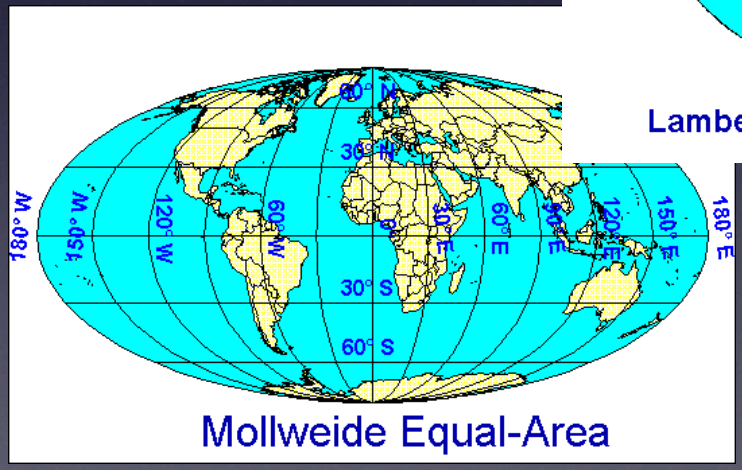




Lambert Azimuthal Equal Area



Sinusoidal Equal-Area



Mollweide Equal-Area

http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj_f.html

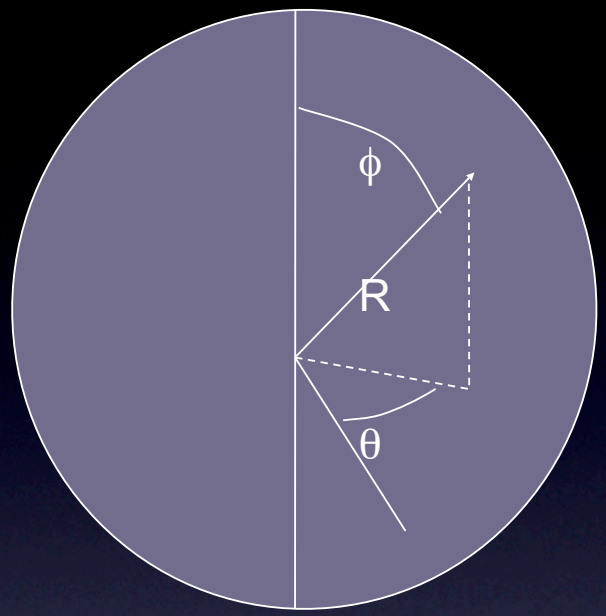
Polar projection

Use longitude/latitude angles:

$$x = \cos\theta \sin\phi$$

$$y = \sin\theta \sin\phi$$

$$z = \cos\phi$$



inverse mapping:

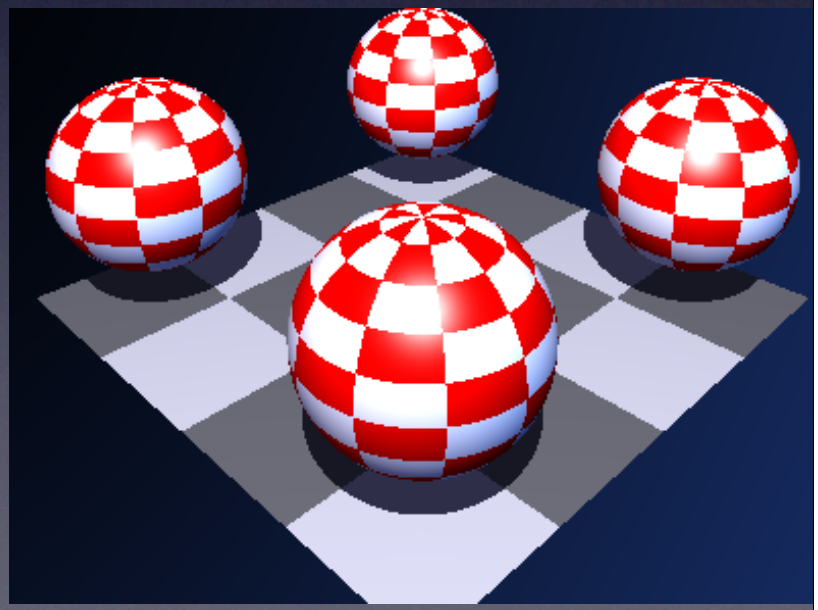
$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\phi = \cos^{-1}\frac{z}{R}$$

$$\theta = \operatorname{atan}2(y,x)$$

$\operatorname{atan}2$: robust atan(y/x) for all quadrants

$$u = \frac{\theta}{2\pi}; \text{if(u<0)u+=1}, v = 1 - \frac{\theta}{\pi}, w = R$$



Arbitrary position/orientation

$\vec{Z}$: vector to north pole

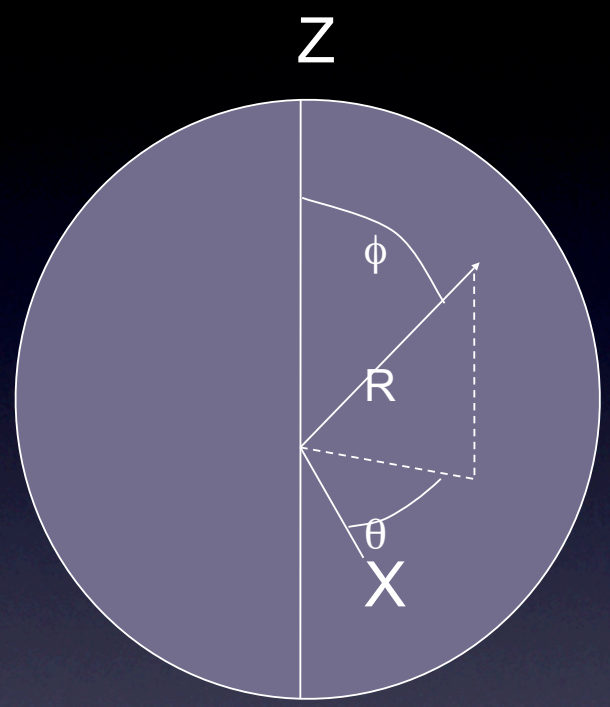
$\vec{X}$: vector to seam

$$\vec{Y} = \vec{Z} \times \vec{X}$$

$$x' = \left(\vec{P} - \vec{C}\right) \cdot \vec{X}$$

$$y' = \left(\vec{P} - \vec{C}\right) \cdot \vec{Y}$$

$$z' = \left(\vec{P} - \vec{C}\right) \cdot \vec{Z}$$



$$x' = \cos\theta \sin\phi$$

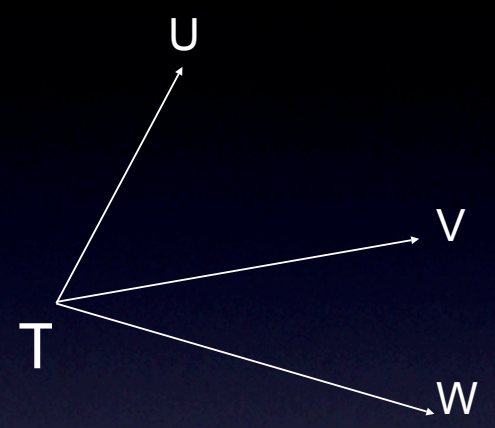
$$y' = \sin\theta \sin\phi$$

$$z' = \cos\phi$$

inverse mapping similar

Linear projection

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} U_x & V_x & W_x \\ U_y & V_y & W_y \\ U_z & V_z & W_z \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

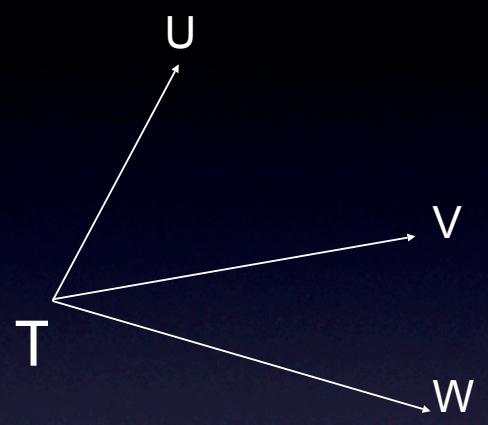


inverse mapping:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} U_x & V_x & W_x \\ U_y & V_y & W_y \\ U_z & V_z & W_z \end{bmatrix}^{-1} \begin{bmatrix} x - T_x \\ y - T_y \\ z - T_z \end{bmatrix}$$

Linear special case

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$



inverse mapping:

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} x - T_x \\ y - T_y \\ z - T_z \end{bmatrix}$$

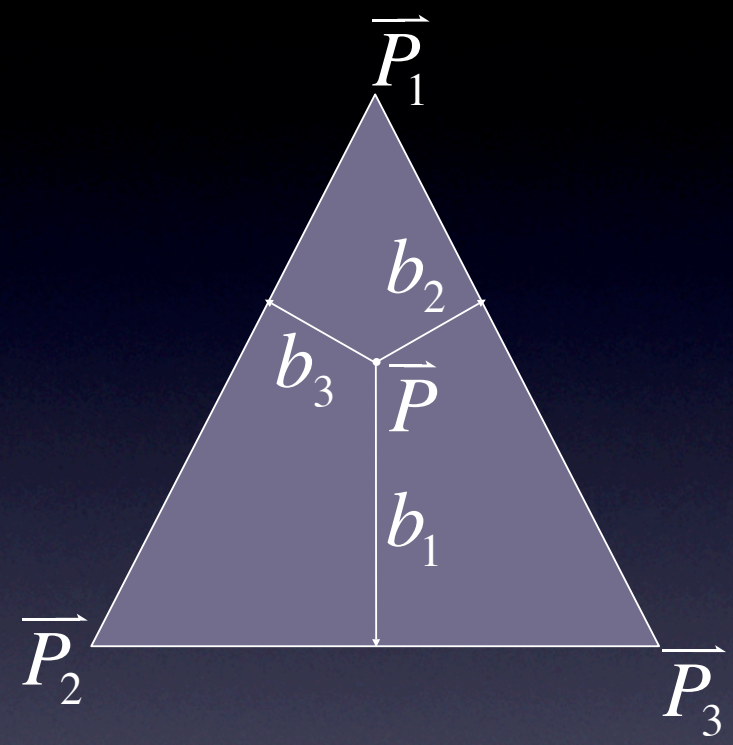
Barycentric coordinates

$$0 \le b_1, b_2, b_3 \le 1$$

$$b_1 + b_2 + b_3 = 1$$

$$\overrightarrow{P} = b_1\overrightarrow{P_1} + b_2\overrightarrow{P_2} + b_3\overrightarrow{P_3}$$

$$= b_1\overrightarrow{P_1} + b_2\overrightarrow{P_2} + \left(1 - b_1 - b_2\right)\overrightarrow{P_3}$$



$$\mathrm{u} = b_1$$

$$v = b_2$$

$$w = 0$$

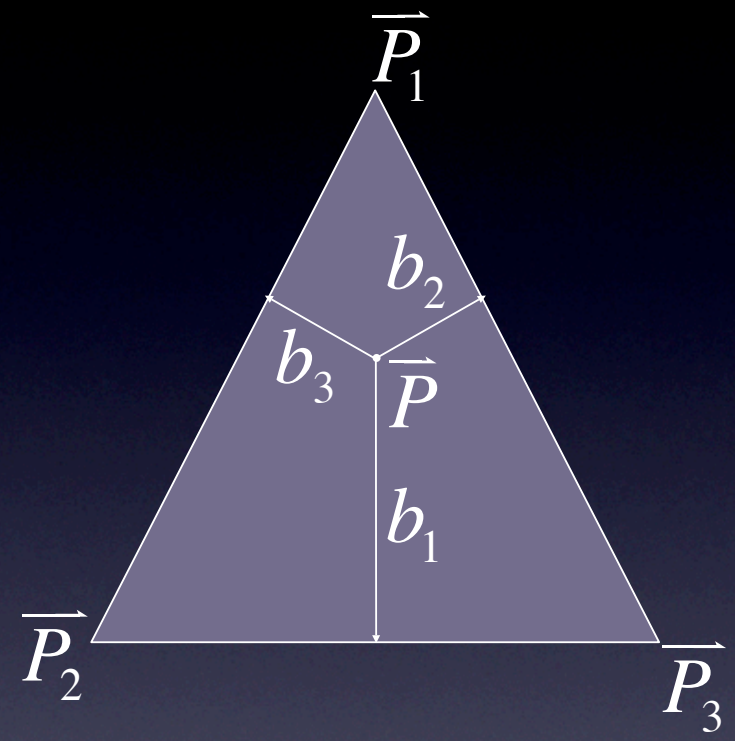
Specified texture coordinates

$$0 \le b_1, b_2, b_3 \le 1$$

$$b_1 + b_2 + b_3 = 1$$

$$\overrightarrow{P} = b_1\overrightarrow{P_1} + b_2\overrightarrow{P_2} + b_3\overrightarrow{P_3}$$

$$= b_1\overrightarrow{P_1} + b_2\overrightarrow{P_2} + \left(1 - b_1 - b_2\right)\overrightarrow{P_3}$$



$$\mathrm{u} = b_1u_1 + b_2u_2 + \left(1 - b_1 - b_2\right)u_3$$

$$v = b_1v_1 + b_2v_2 + \left(1 - b_1 - b_2\right)v_3$$

$$w = b_1w_1 + b_2w_2 + \left(1 - b_1 - b_2\right)w_3$$

Other coordinate transforms

- Any of those spherical projections
- Could transform u/v on the disc or ring (similar to cylindrical)
- Conical projections
- Many more

Implementation choices

- Where do these coordinate transforms go?
  - Object? Might want linear or cylindrical mapping on a sphere
  - Material? Material doesn't know about coordinates
  - Somewhere else?

Implementation tradeoffs

- Object: different sphere subclasses for different UV mappings
- Object: multiple inheritance
- Material: similar tradeoffs
- Somewhere else: redundant data?
- Somewhere else: multiple inheritance

What Steve does

- Add to primitive base class:

```
TexCoordMapper* tcmapper;
void setTexCoordMapper(TexCoordMapper*);
TexCoordMapper* getTexCoordMapper();
```

- Sphere is multiply inherited and default tcmapper is “this” pointer
- Objects with no inherent uv space default to linear tcmapper (xyz == uvw)
- Default space can be overridden in scene

Image Textured Lambertian Shading

Compute hit position ($\vec{P} = \vec{O} + t\vec{V}$)

Call primitive to get tcmapper

Call tcmapper to get UVW coordinates ($\vec{T}$)

Interpolate surface color from image using $\vec{T}$ scaled to image resolution

Call primitive to get normal ($\vec{N}$) (normalized)

$costheta = -\vec{N} \cdot \vec{V}$

$if(costheta < 0)$

  normal =-normal

Color light = scene.ambient*Ka

foreach light source

  get C_L and $\vec{L}$

  dist=$\|\vec{L}\|, \vec{L_n} = \frac{\vec{L}}{\|\vec{L}\|}$

  $cosphi = \vec{N} \cdot \vec{L_n}$

  if($cosphi > 0$)

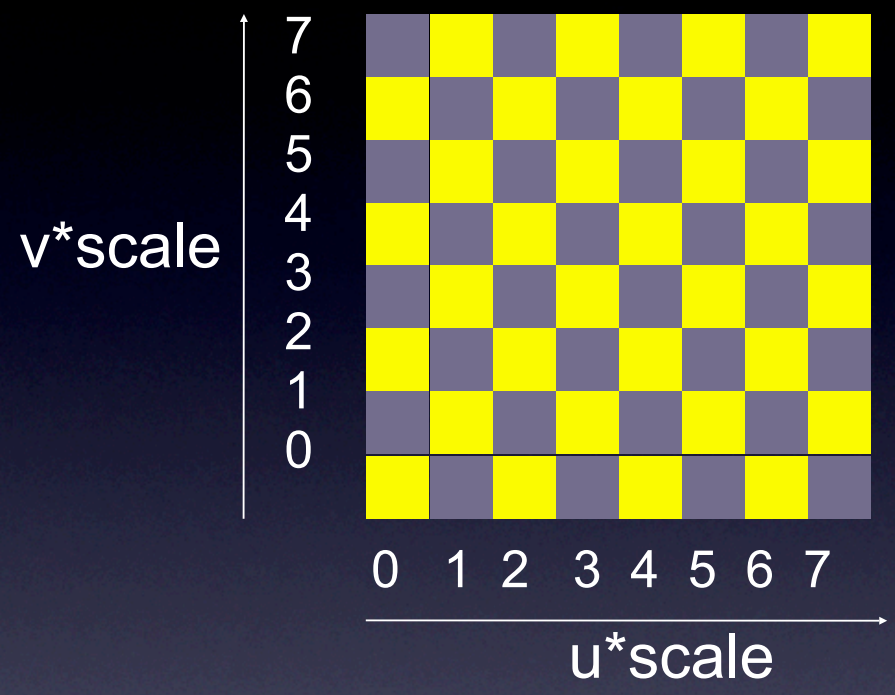
    if(!intersect with $0 < t < dist$)

      light += $C_L * (Kd * cosphi)$

result=light*surface color

Checkerboard texture

$$i_1 = (\text{int})(u * scale)$$
$$i_2 = (\text{int})(v * scale)$$



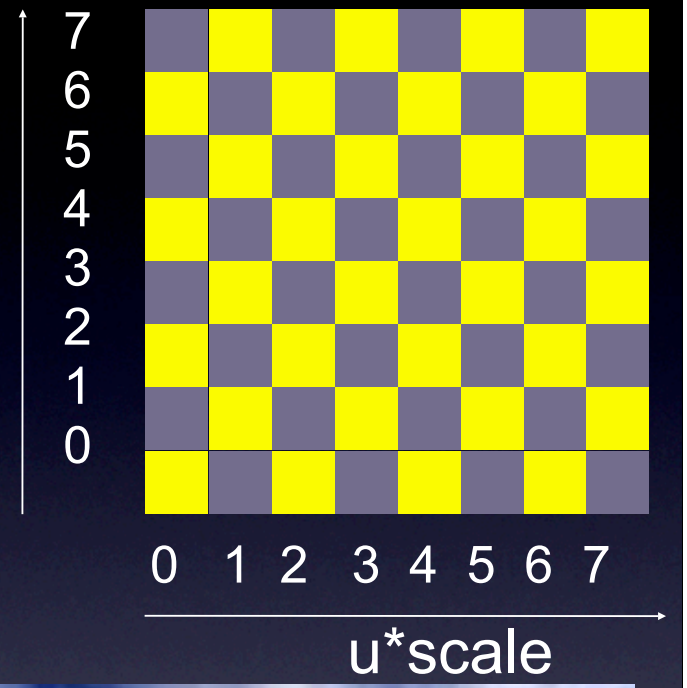
$$cell = (i1 + i2)\%2$$

$$if\,(cell = 0)\ \text{use color1}$$

else use color2

3D Checkerboard texture

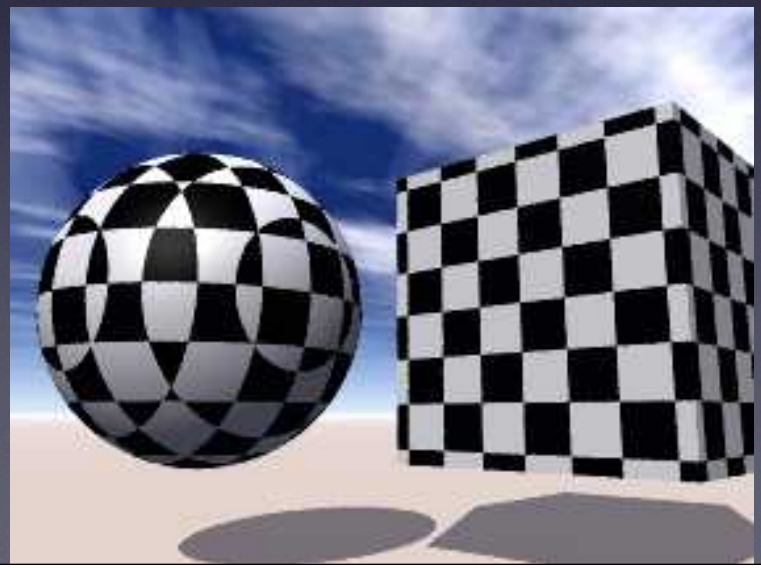
$$i_1 = (\text{int})(u * scale)$$
$$i_2 = (\text{int})(v * scale)$$
$$i_3 = (\text{int})(w * scale)$$



$$cell = (i1 + i2 + i3)\%2$$

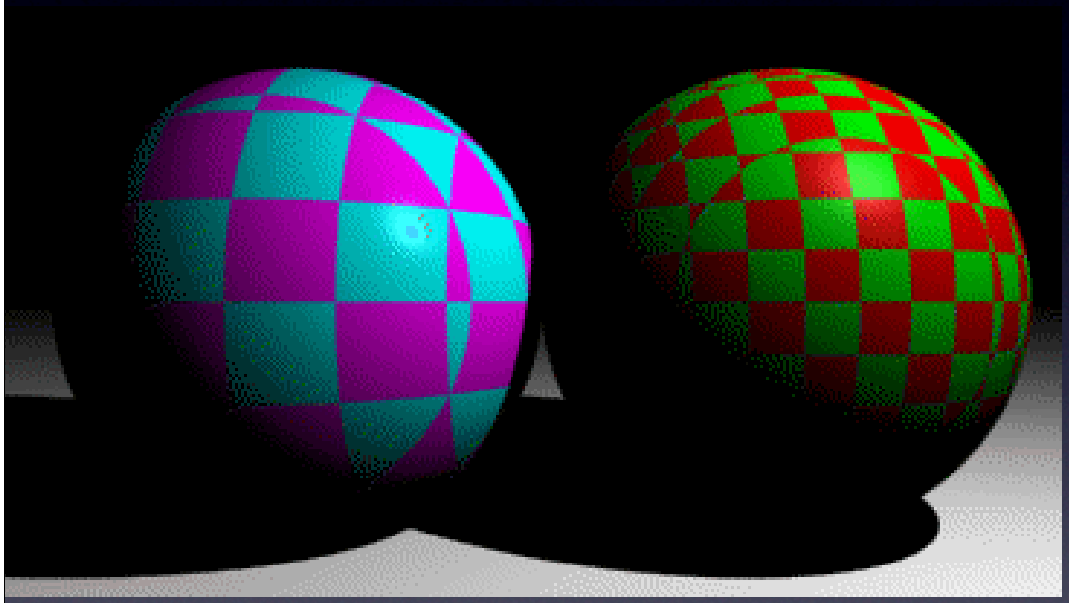
$if(cell = 0)$ use color1

else use color2



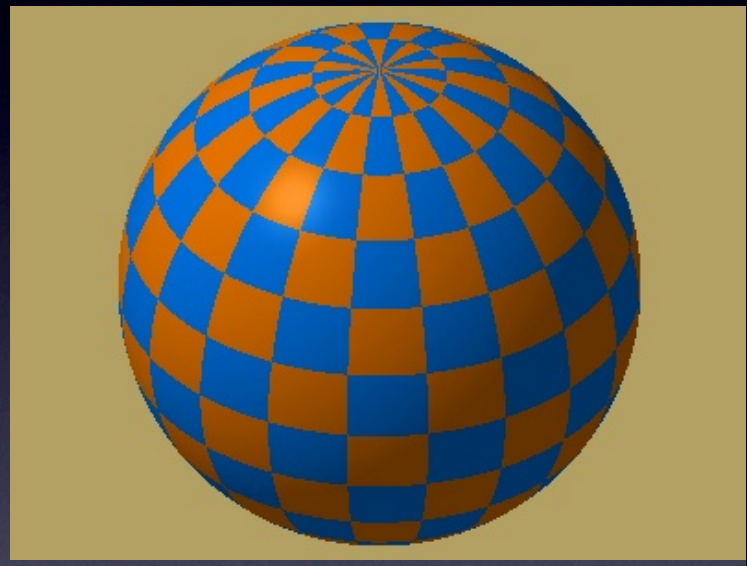
http://www.chez.com/jrlivenais/vdesprit/tut_index/check_sphere/check_sphere_eng.htm

Checkboard comparisons



3D checker
planar mapping

http://courses.dce.harvard.edu/~cscie234/projects/Slocum/checker-spheres-4x4-jitt-1.gif



2 D (or 3D) checker
spherical mapping

http://cgg.ms.mff.cuni.cz/~pepca/lectures/textures/sample/checker.jpg

Floor tile texture

$$s = u * scale - (\text{int})(u * scale)$$

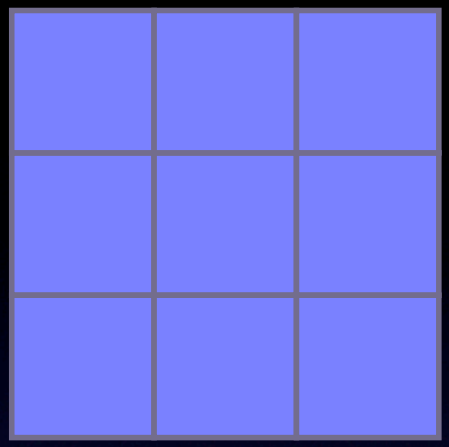
$$t = v * scale - (\text{int})(v * scale)$$

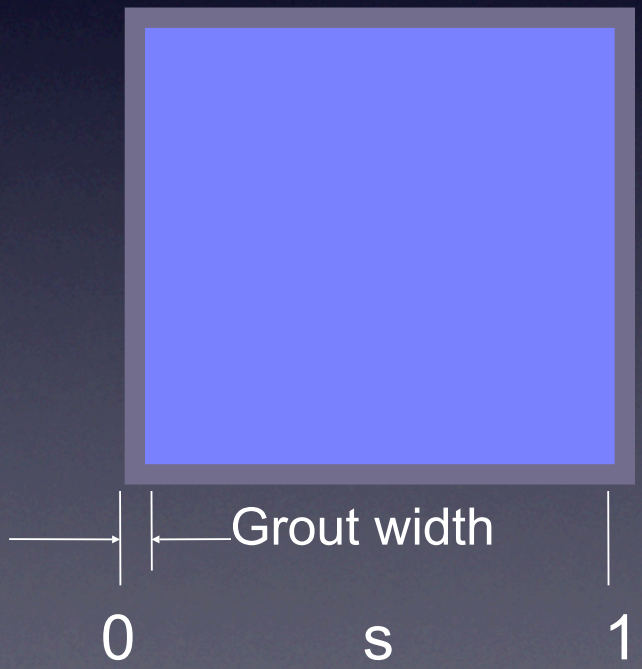
$$if\,(s < grout\_width \;\|\; t < grout\_width)$$

$$\quad color = grout\ color$$

$$else$$

$$\quad color = tile\ color$$





Brick texture

$ti = (\text{int})(v * vscale)$ // get brick row

$u' = u * uscale - (ti\%2) * 0.5$ // shift column for odd rows

$s = u' - (\text{int})u'$ // brick column fraction

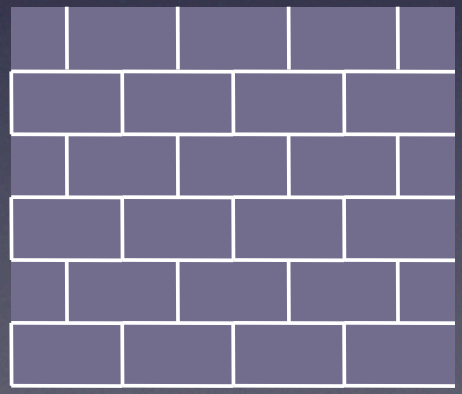
$t = v * vscale - ti$ // brick row fraction

$if\,(s < mortar\_width \,||\, t < mortar\_width)$

$\quad color = mortar\_color$

$else$

$\quad color = brick\_color$



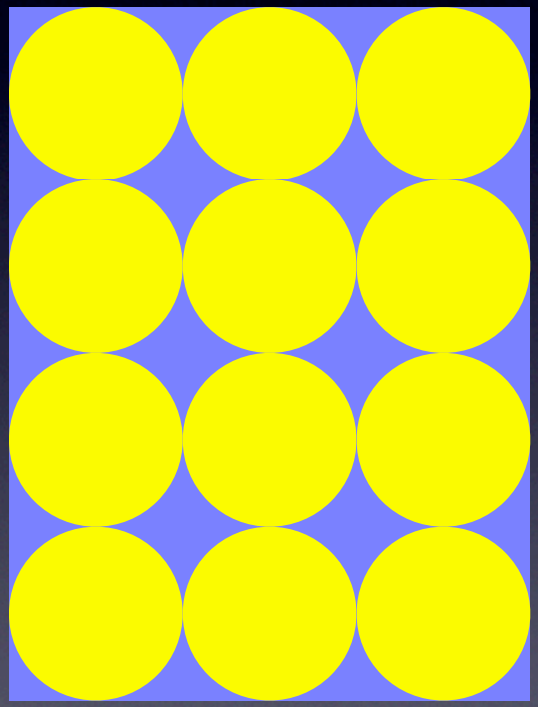
More flexible implementation

- Instead of choosing between two (or more) colors, choose between two materials

```
if(cell == 0)
   matl1->shade(…);
else
   matl2->shade(…);
```

Other simple procedural textures

- Hexagonal mapping
- Cutout shapes
- Use your imagination!

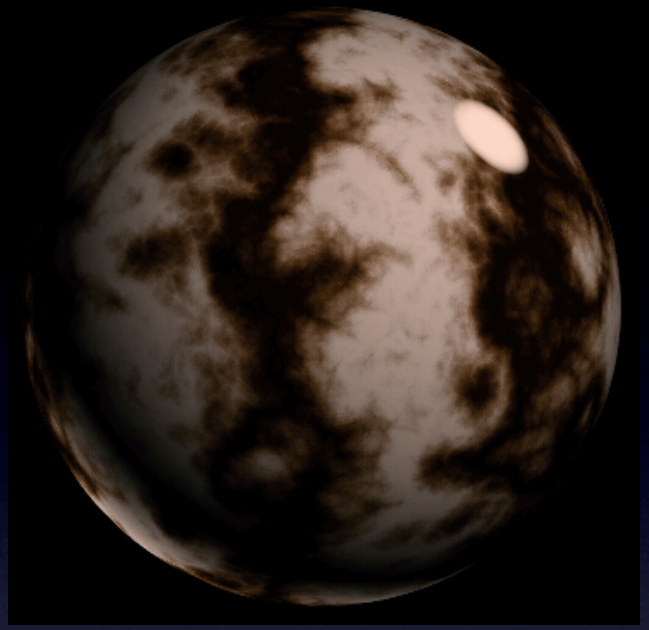


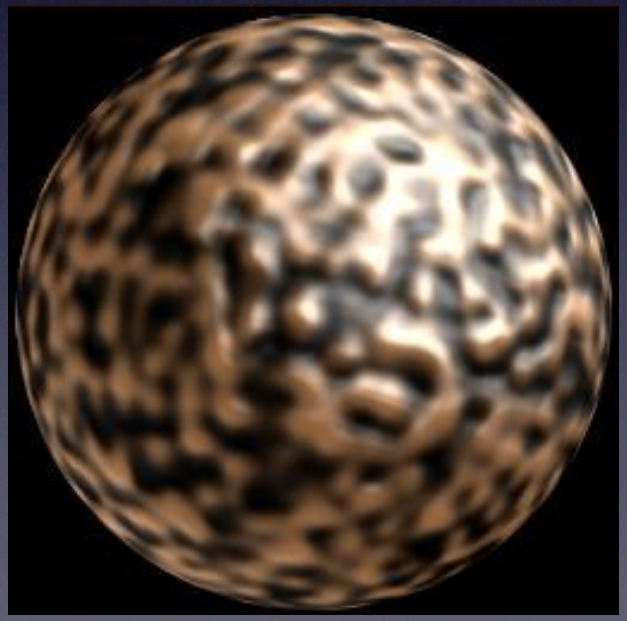
Solid textures

- Many objects are created out of solid materials
- Texture is three-dimensional
- Surface texture is cutaway of the 3D texture space
- Examples:
  - Wood
  - Marble
  - Granite

Perlin noise

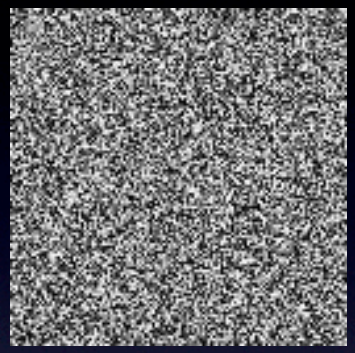
- A pseudorandom method for making natural textures
- Appeared in Siggraph 1985 (preview in 1984)
- Ken Perlin won an academy award (technical achievement award) in 1997
- More info: http://noisemachine.com
- Improved in 2002:
  - http://mrl.nyu.edu/~perlin/noise



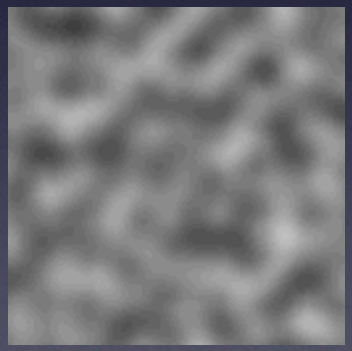


Perlin noise

- Need a function that:
  - Looks random
  - Has fixed frequency content
  - Is coherent (value changes smoothly from one point to another)
- Solution: smooth random values on a regular grid



Incoherent

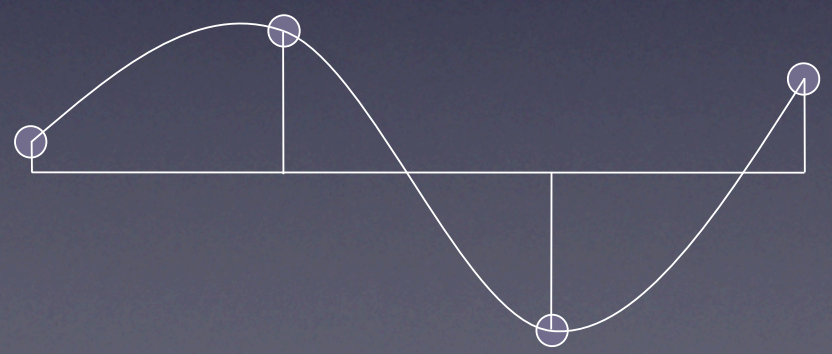


Coherent

http://www.robo-murito.net/code/perlin-noise-math-faq.html

Value noise function

- Idea: place random values on a grid lattice
- Interpolate between them (spline)
- Spline is expensive in 3D



Perlin noise (gradient noise)

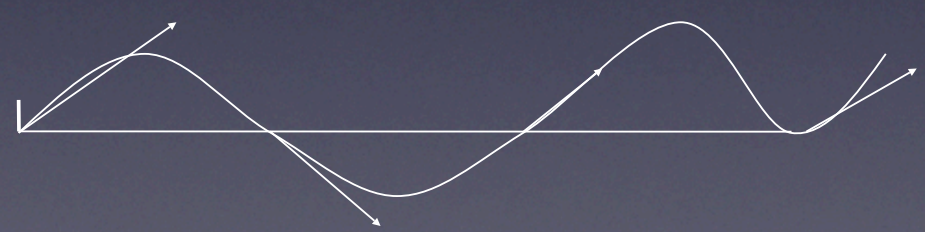
- Idea: place random gradients on a grid lattice
- Weighted interpolation of gradient values

$$0 \le x \le 1$$

$$g_0 = G_0 x, g_1 = G_1 (1 - x)$$

$$noise(x) = lerp(g_0, g_1, ease(x))$$

Note: $noise(0) = 0, noise(1) = 0$

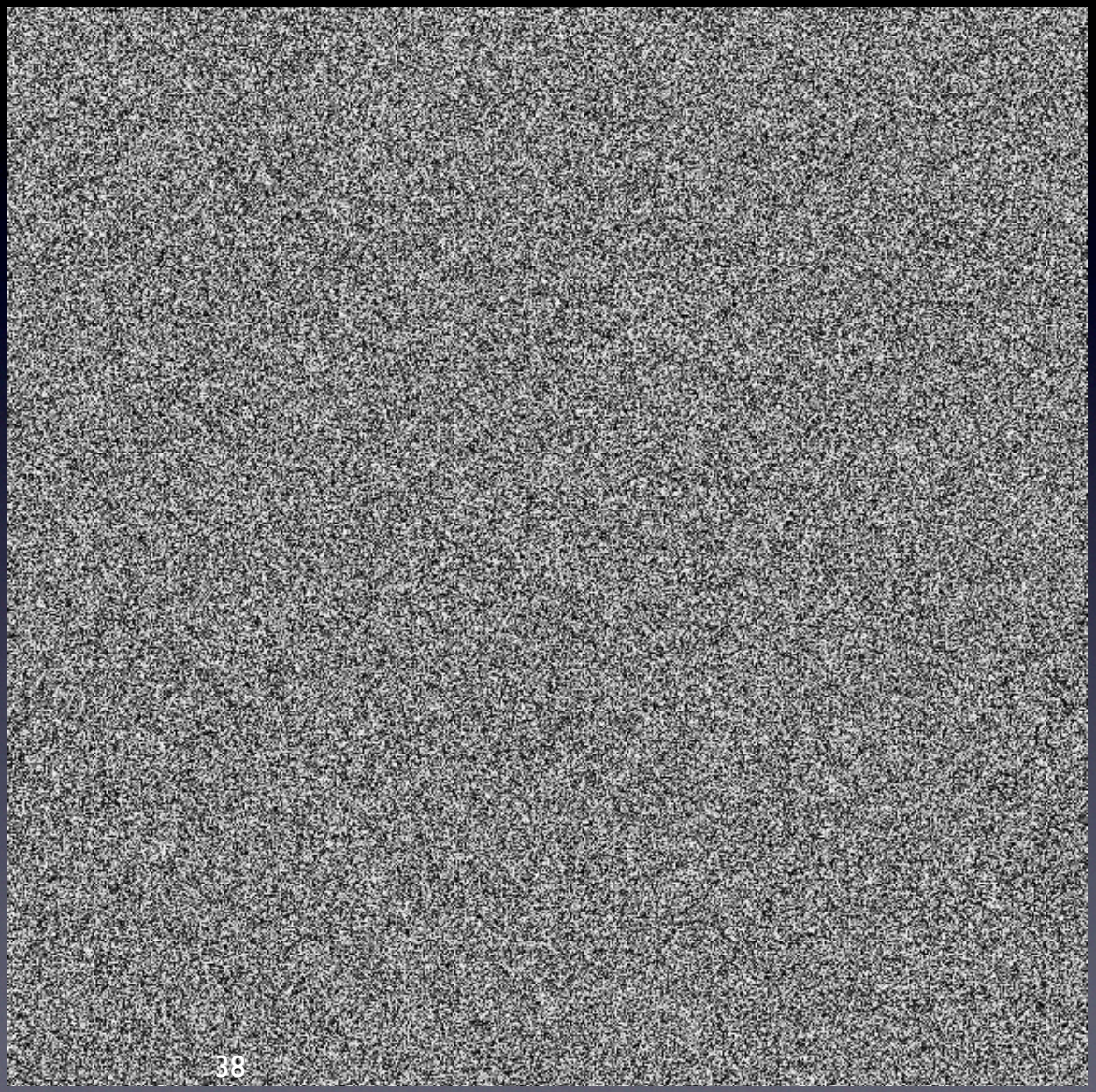


Noise

- Properties of good noise functions
  - Repeatable
  - Bandlimited
  - Stays within known limits
  - Isotropic

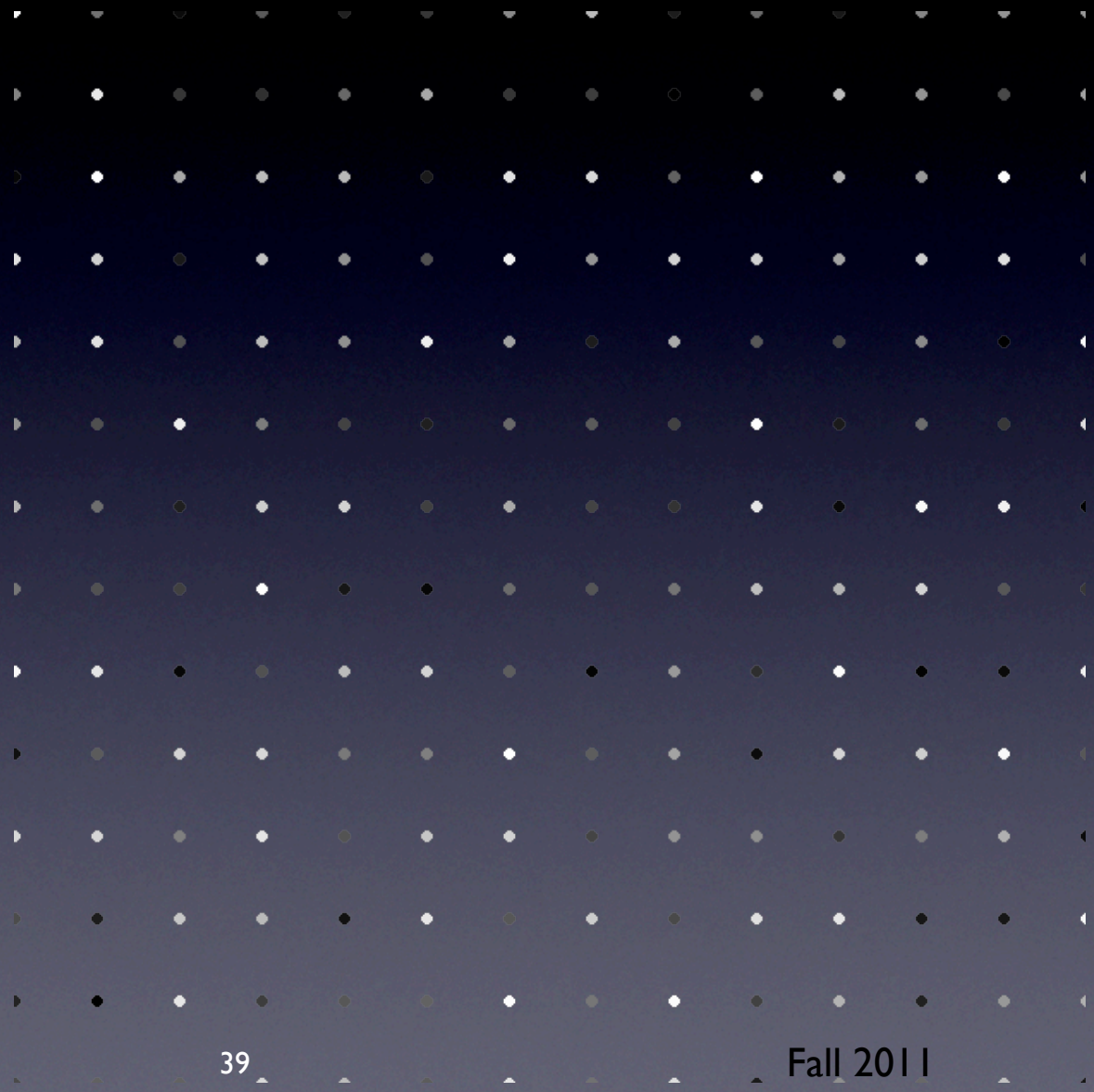
Value Noise

Take a continuous random process. (white noise)



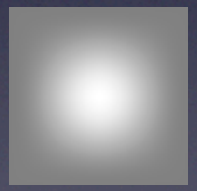
Value Noise

Sample it.



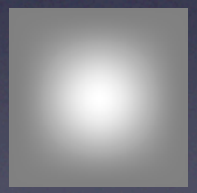
Value Noise

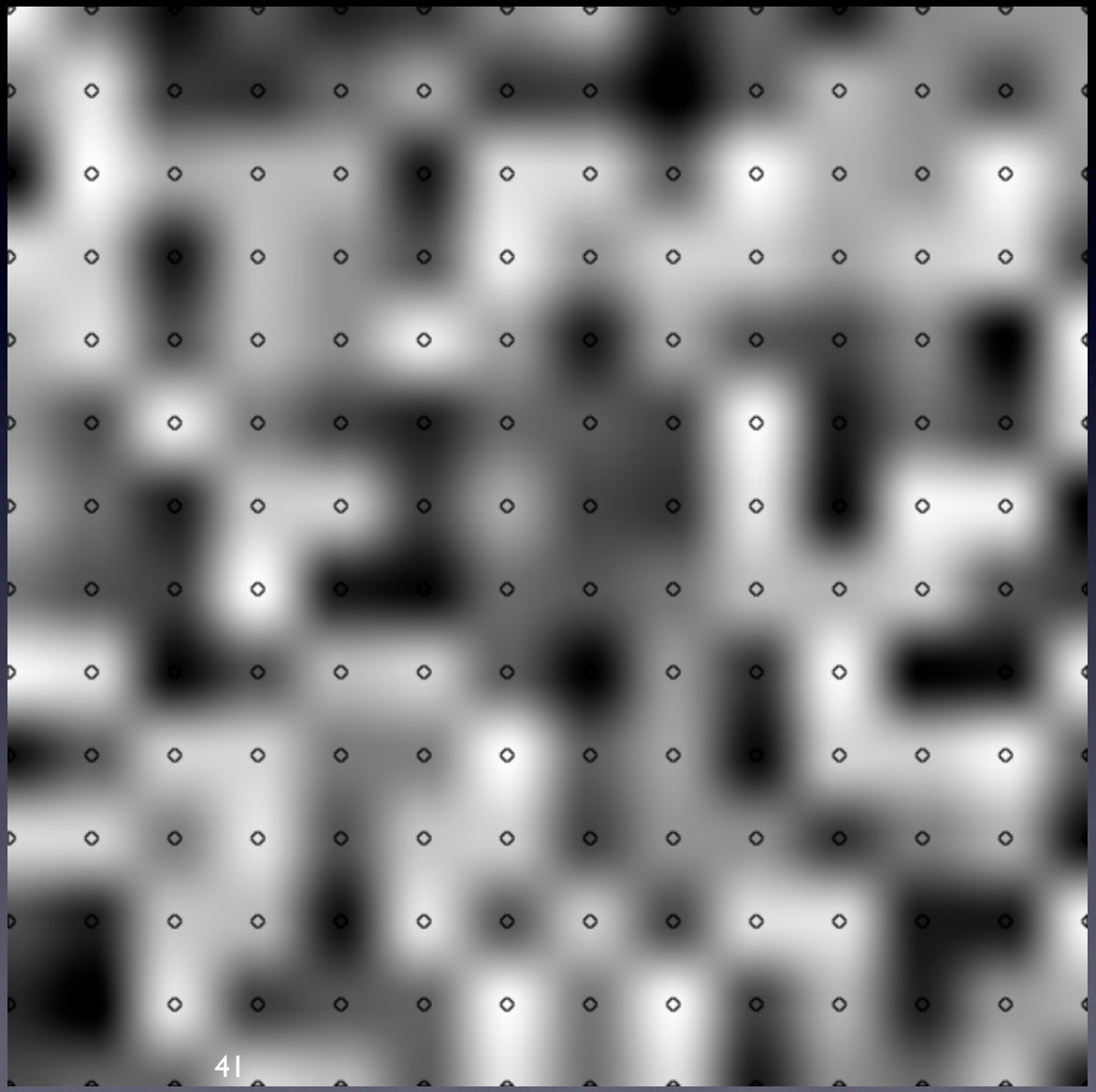
Convolve the samples with a reconstruction filter.



Value Noise

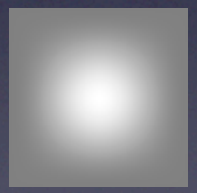
This produces basic value noise.





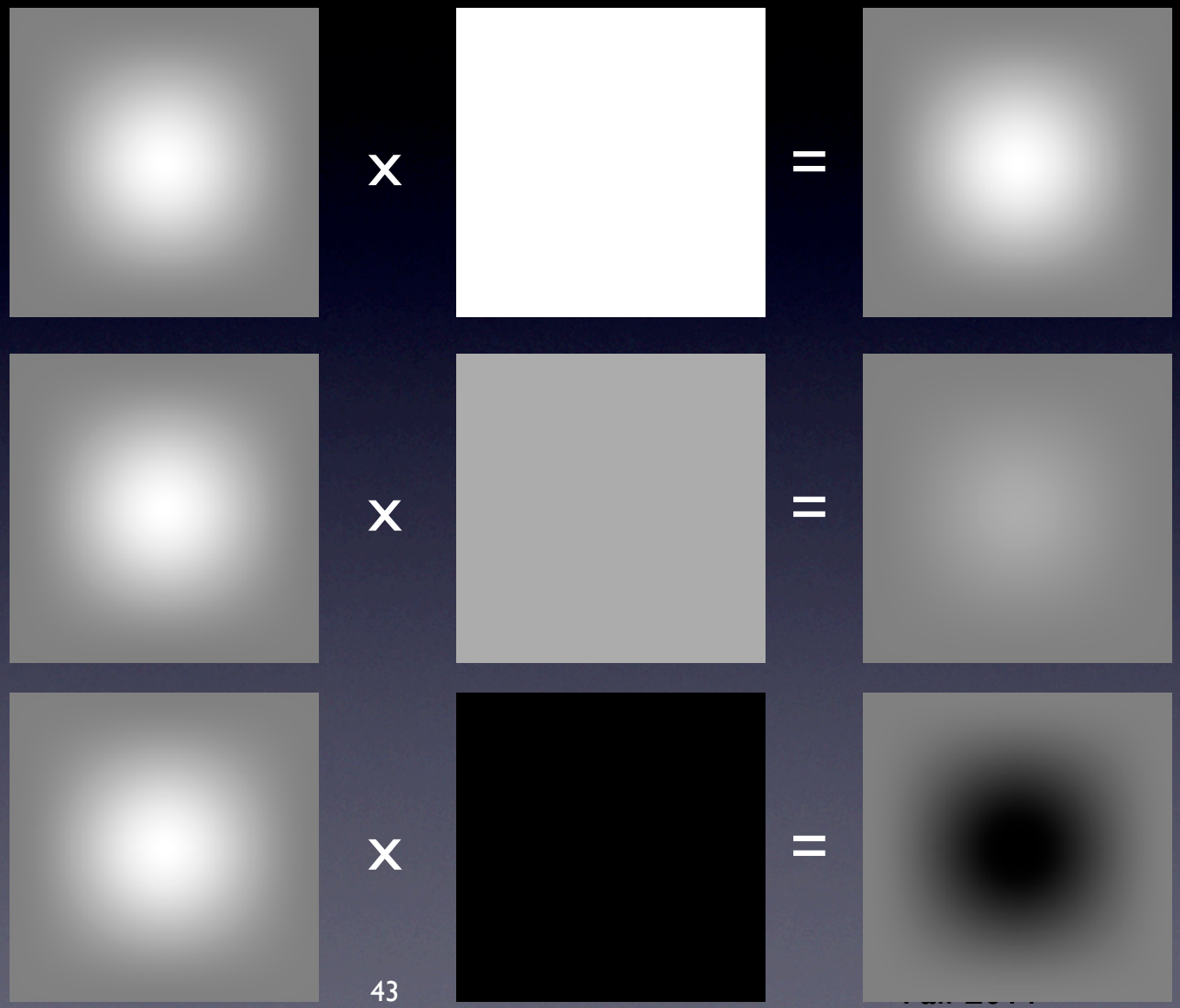
Value Noise

This produces basic value noise.



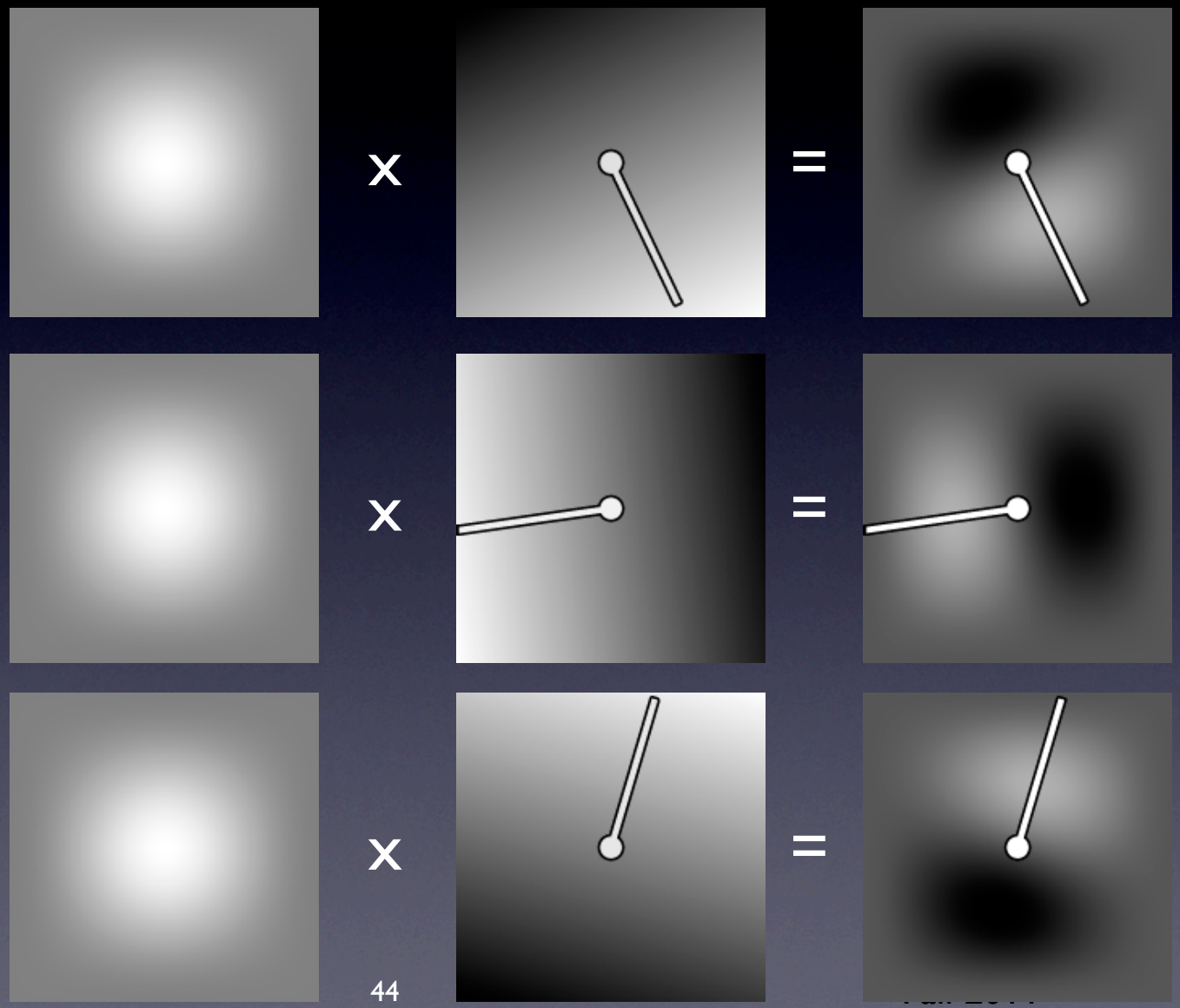
Perlin's Insight

Value noise just multiplies the kernel with a simple constant.



Perlin's Insight

Suppose instead that we multiplied the kernel with a gradient?

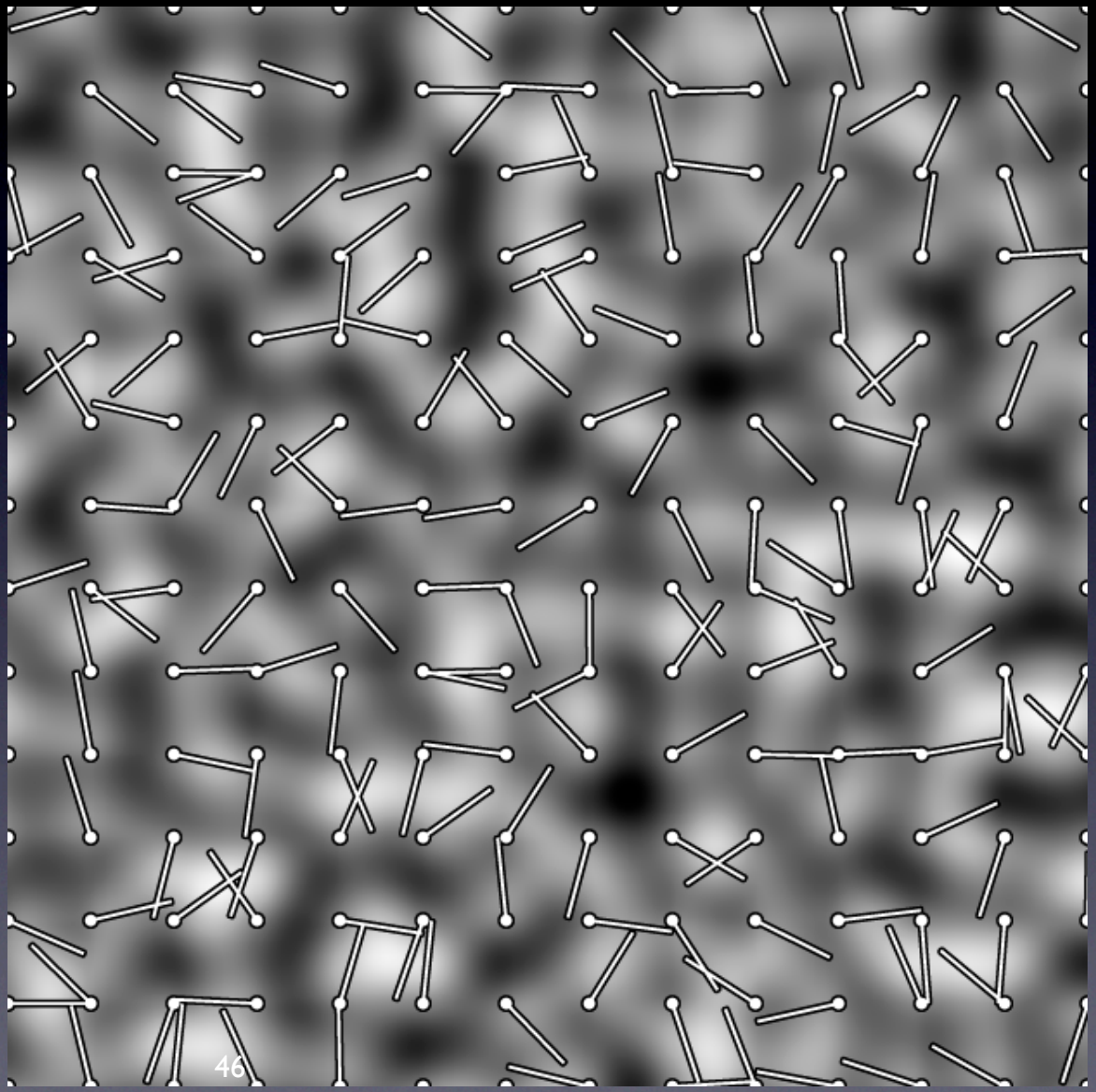


Perlin Noise

Instead of random intensities at each point, use random unit vectors.

Perlin Noise

The result is Perlin noise.



Perlin Noise

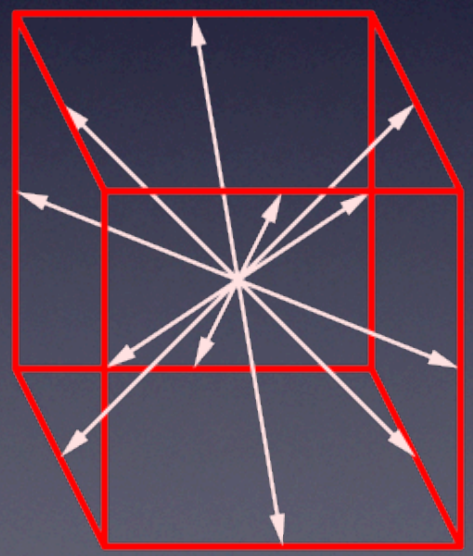
The result is Perlin noise.

Classic Implementation

```
// P[n] = permuation table: numbers 0 through n-1, shuffled
// G[n] = gradients: random unit vectors
float noise(point S)
{
    I = floor(S)
    F = S - I
    ease = 3F^2 - 2F^3
    for each corner, C, of the containing cube:
        hash = P[C_x + P[C_y + P[C_z mod n] mod n] mod n]
        dot = G[hash] . (S-C)
    return trilinear(dot, ease)
}
```

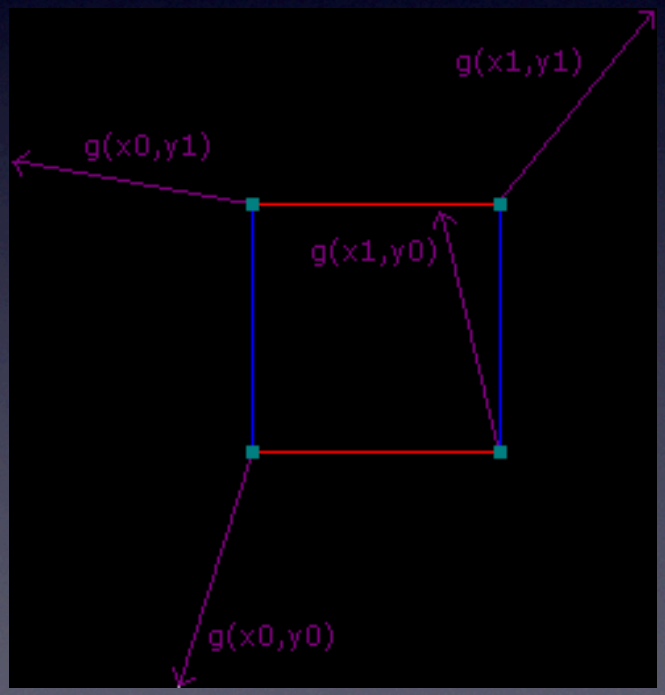
“Improved” Perlin

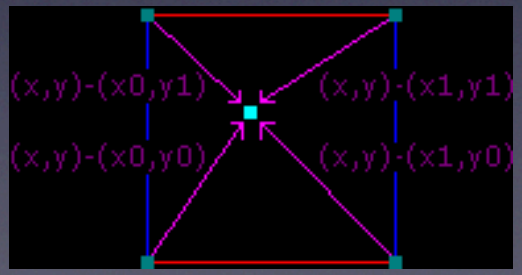
- Higher order ease function: $6F^5 - 15F^4 + 10F^3$
- Evenly distribute gradients: use just 12 vectors to center of cube edges



2D perlin noise

- Sum over each corner: weighted gradient dot vector to point





http://www.robo-murito.net/code/perlin-noise-math-faq.html

Ease curve

- Get rid of discontinuities at grid boundaries
- Interpolate with hermite polynomial

Linear

$2t^3 - 3t^2$

First derivative == 0 at t=0 and t=1

Faster way

- Randomly choose a gradient vector based on a hash function:
  - Precompute table of permutations P[n]
  - Precompute table of gradients G[n]
  - G = G[(i + P[(j + P[k]) mod n]) mod n]

2002 implementation

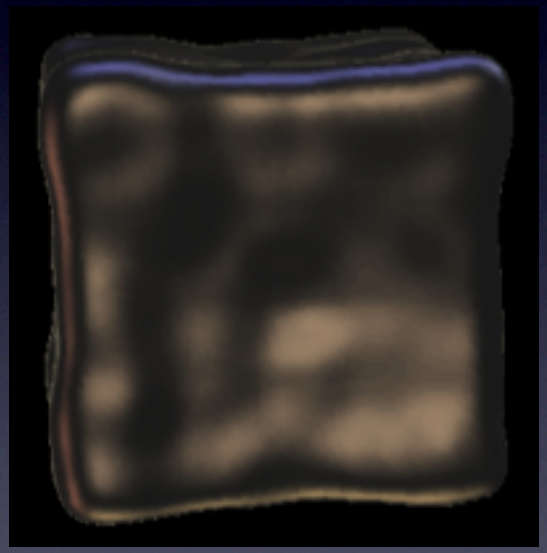
- Randomness comes from permutation and random gradients
- Random gradients are not necessary
- Use gradients to edges of a cube:
- [0 1 1] [0 1 -1] [0 -1 1] [0 -1 -1]
- [1 0 1] [1 0 -1] [-1 0 1] [-1 0 -1]
- [1 1 0] [1 -1 0] [-1 1 0] [-1 -1 0]
- Use better ease function:

$$6t^5 - 15t^4 + 10t^3$$

Comparison

- 2002 implementation is a little faster and looks better

Original



Improved

Easy way

- Perlin's 2002 reference implementation (Java)
- http://mrl.nyu.edu/~perlin/noise/

Improved Noise

- Better gradient noise
  - Software implementation
  - http://www.cs.utah.edu/~aek/research/noise.pdf
- Hardware-accelerated gradient noise for graphics
  - VLSI implementation
  - 4-stage pipeline at 1 GHz
  - www.cs.utah.edu/~aek/research/hwnoise.pdf

Fractal noise

- Perlin noise is (nearly) fixed frequency
  - Defined by the grid
- Fractal noise:

$$turbulence(x) = noise(x) + \frac{1}{2} noise(2x) + \frac{1}{4} noise(4x) + \ldots$$

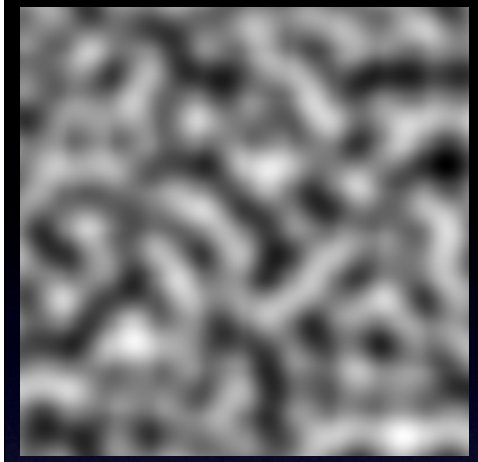
More generally:

$$turbulence(x) = \sum_{i=0}^{octaves} \frac{a^i noise(2^i)}{b^i}$$

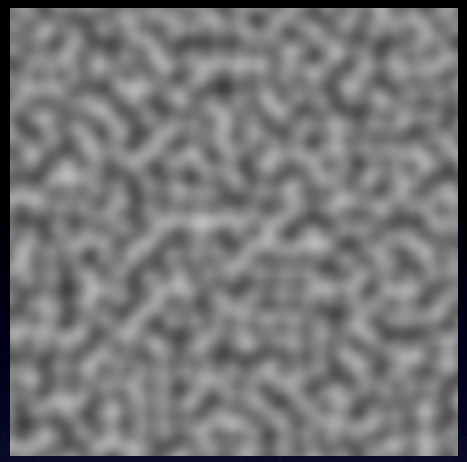
a: lacunarity

b: gain or persistence

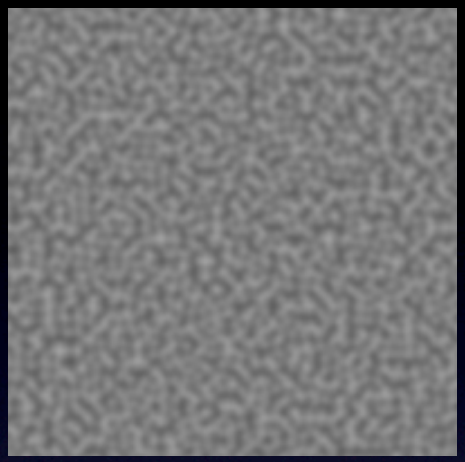
Fractal noise



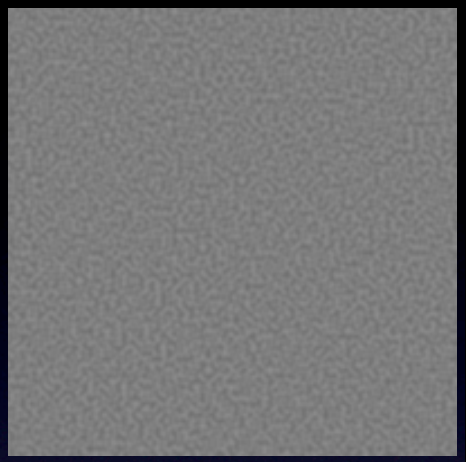
1x



2x

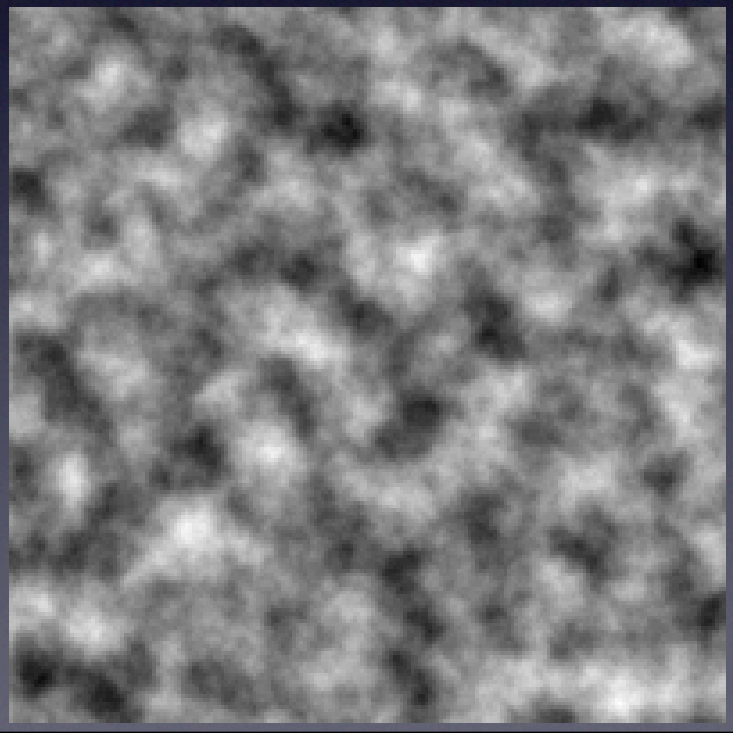


4x



8x

Sum



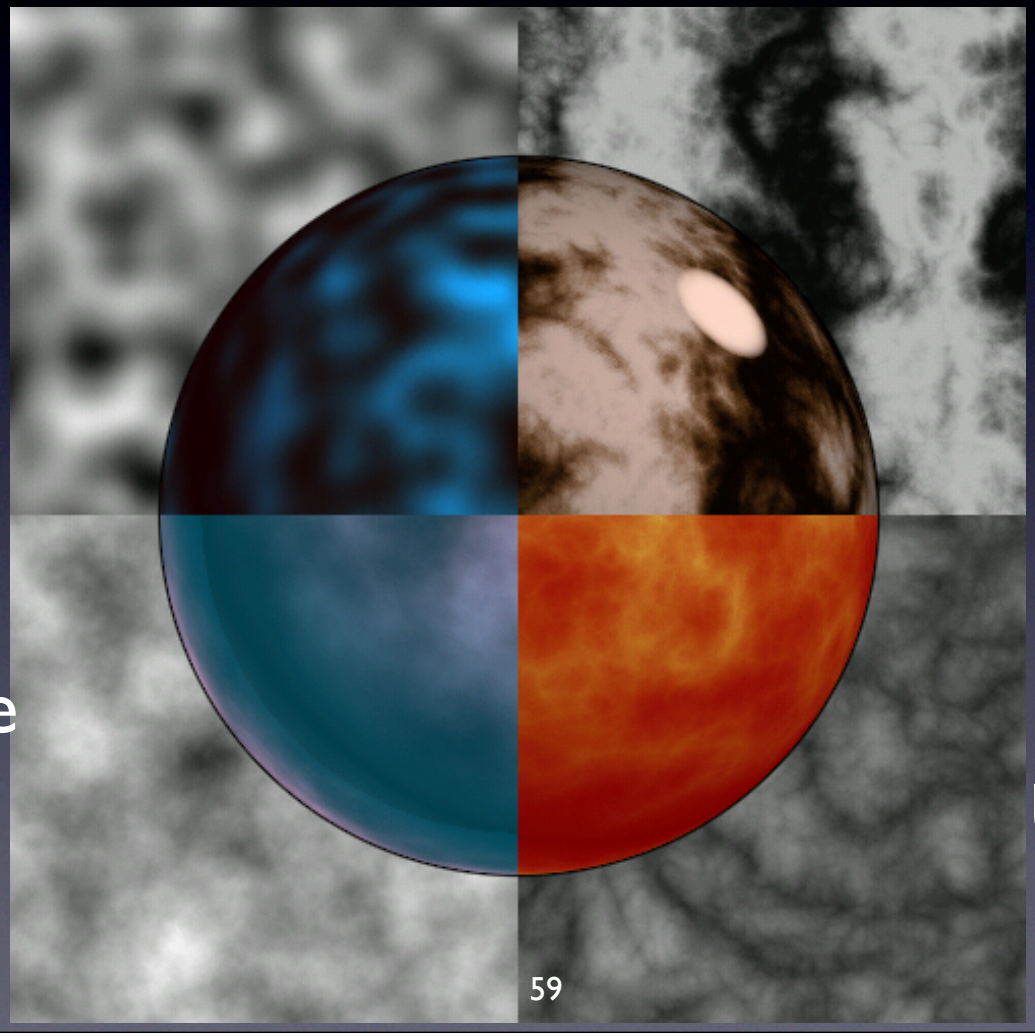
http://astronomy.swin.edu.au/~pbourke/texture/perlin/

Using noise

- Creativity is in using functions of noise or turbulence

Noise

sin(x+turb(x))



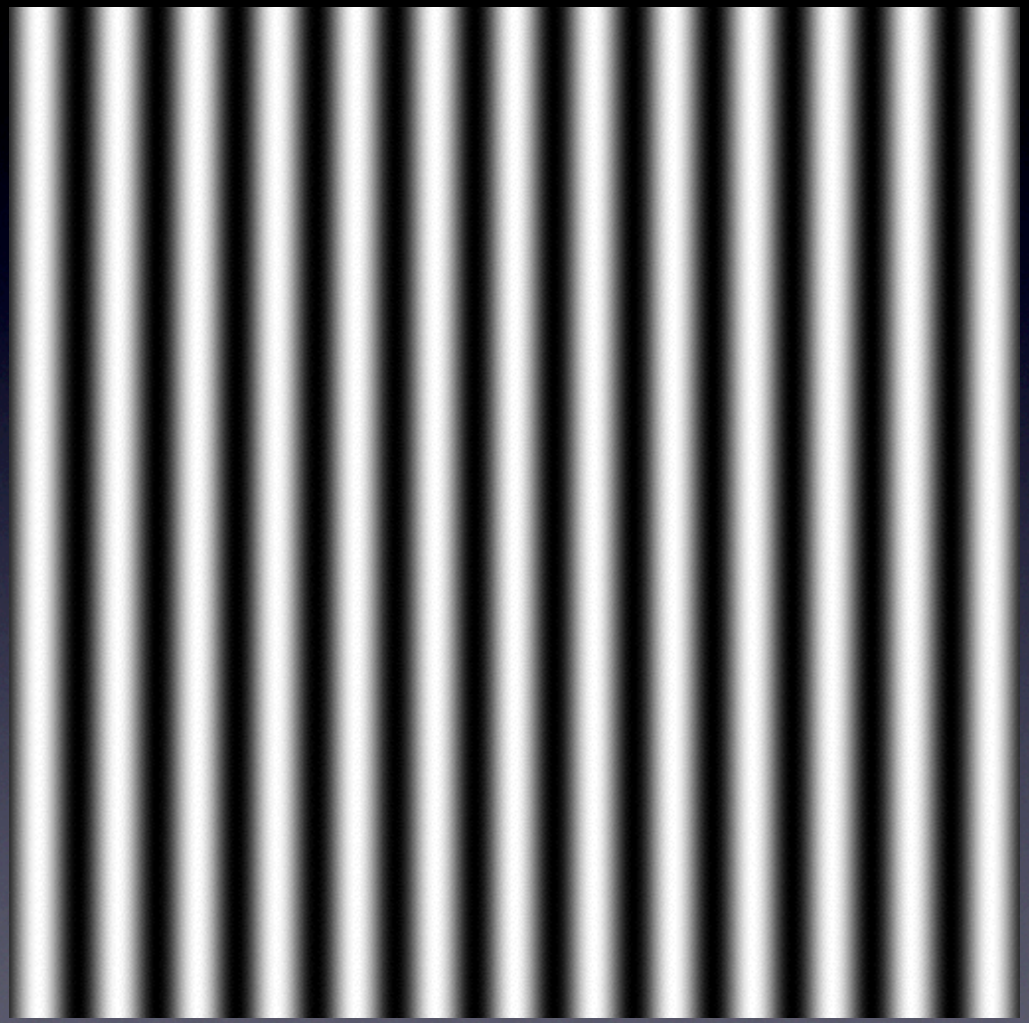
Turbulence

Turbulence Using |noise|

Marble

sin(40x)*0.5+0.5

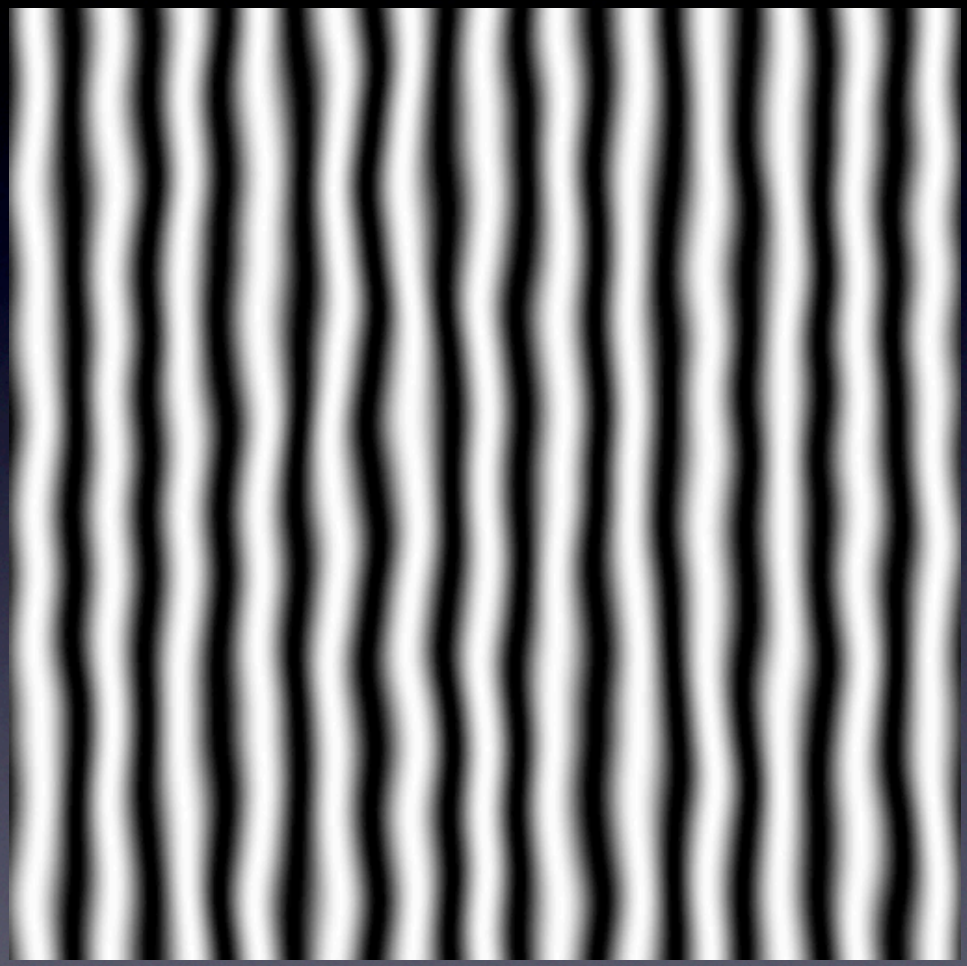
- Swirled veins of molten rock



Marble

- Swirled veins of molten rock

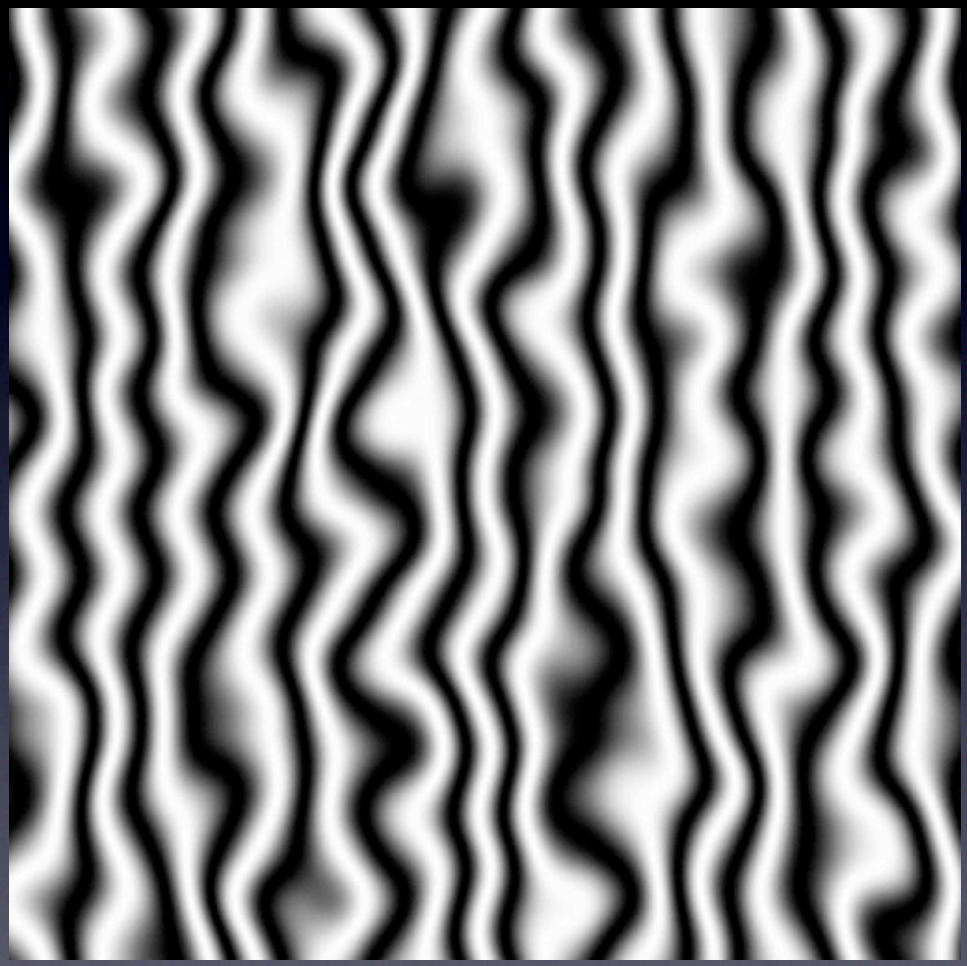
sin(40x+noise(4x))*0.5+0.5



Marble

- Swirled veins of molten rock

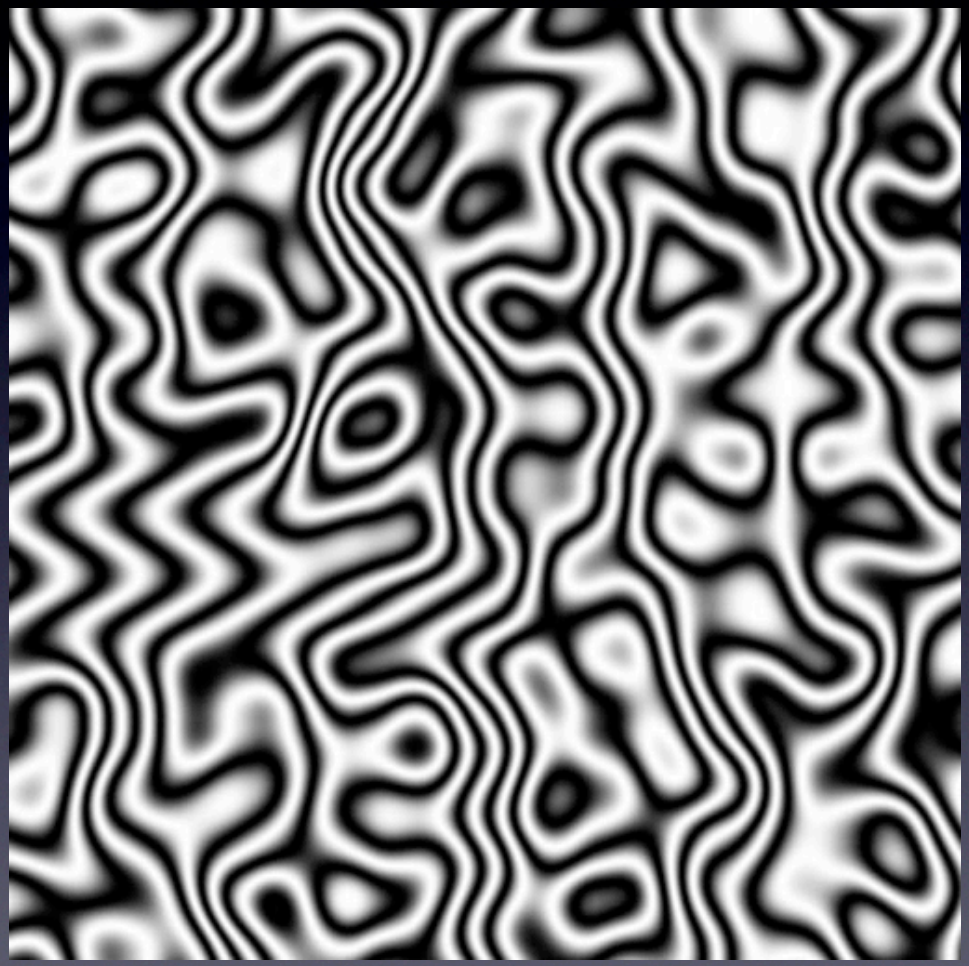
sin(40x+5noise(4x))*0.5+0.5



Marble

sin(40x+10noise(4x))*0.5+0.5

- Swirled veins of molten rock



Marble

sin(40x+16noise(4P)+8noise(8P))*0.5+0.5
=sin(40x+16turbulence(2, 4P, 2, 0.5))*0.5+0.5

- Swirled veins of molten rock

Marble =sin(40x+16turbulence(3, 4P, 2, 0.5))*0.5+0.5

- Swirled veins of molten rock

Marble =sin(40x+16turbulence(4, 4P, 2, 0.5))*0.5+0.5

- Swirled veins of molten rock

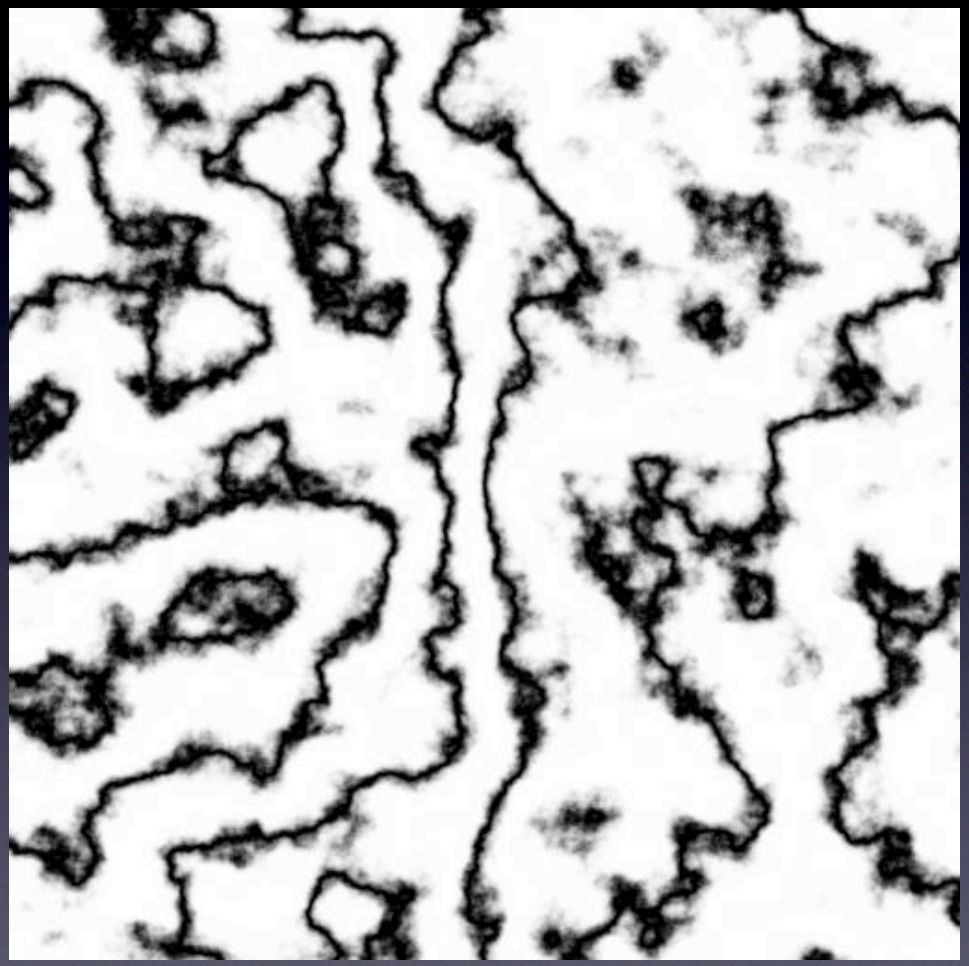
Marble =sin(40x+16turbulence(6, 4P, 2, 0.5))*0.5+0.5

- Swirled veins of molten rock

Marble

=1-(sin(40x+16turbulence(6, 4P, 2, 0.5))*0.5+0.5)^6

- Swirled veins of molten rock

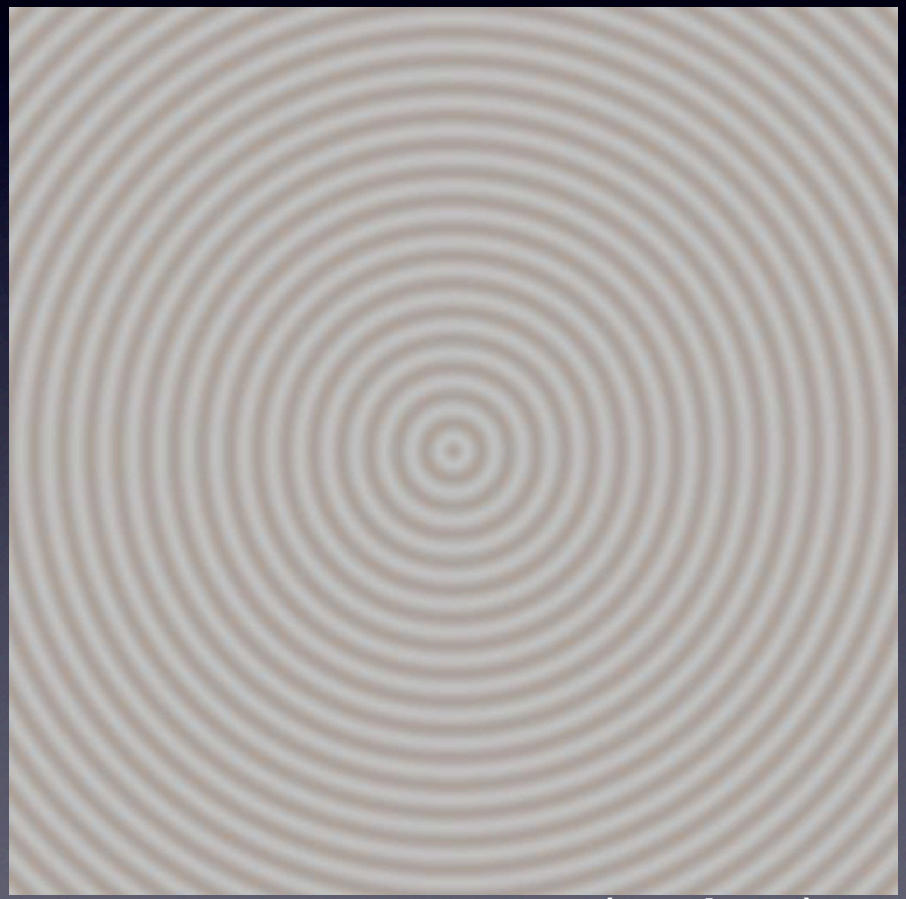


Advanced marble

- Use |noise| instead of noise
  - Makes sharper features
- Use different colors
  - Interpolate between two colors
  - General spline function

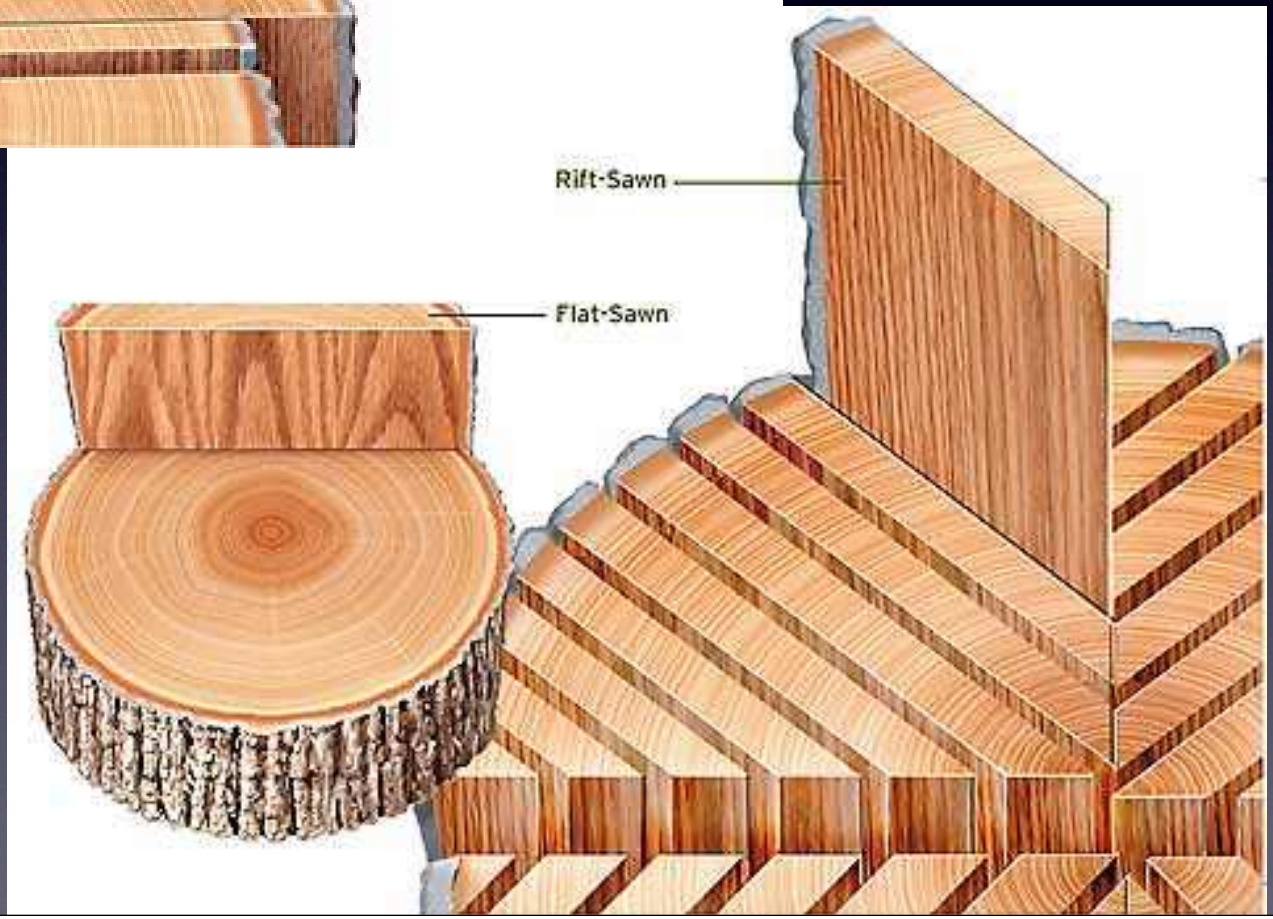
Wood texture

- Trees are concentric cylinders of growth



cos(radius)

Wood cuts:
Quarter-sawn
Flat-sawn
Rift-sawn
Rotary sliced



Hardwood.org

Plain-sawn

Rift-sawn

http://www.wood-veneers.com/

Maple

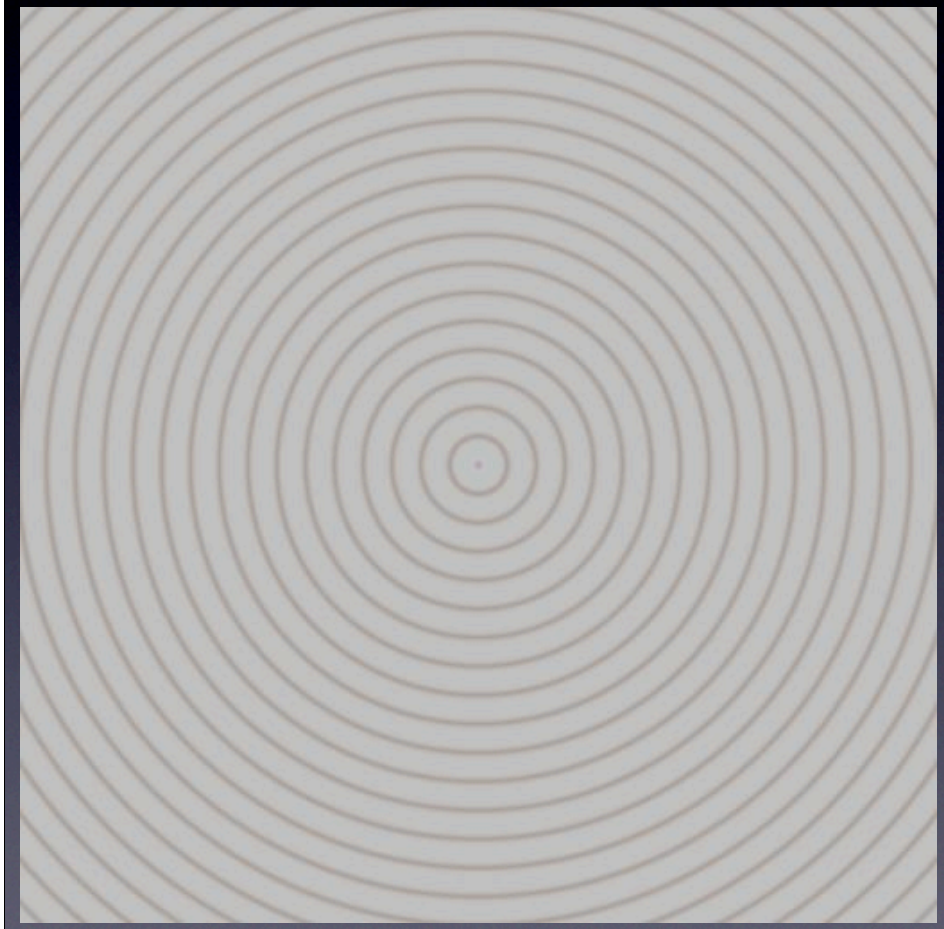
Curly maple

Walnut

http://www.wood-veneers.com/

Wood texture

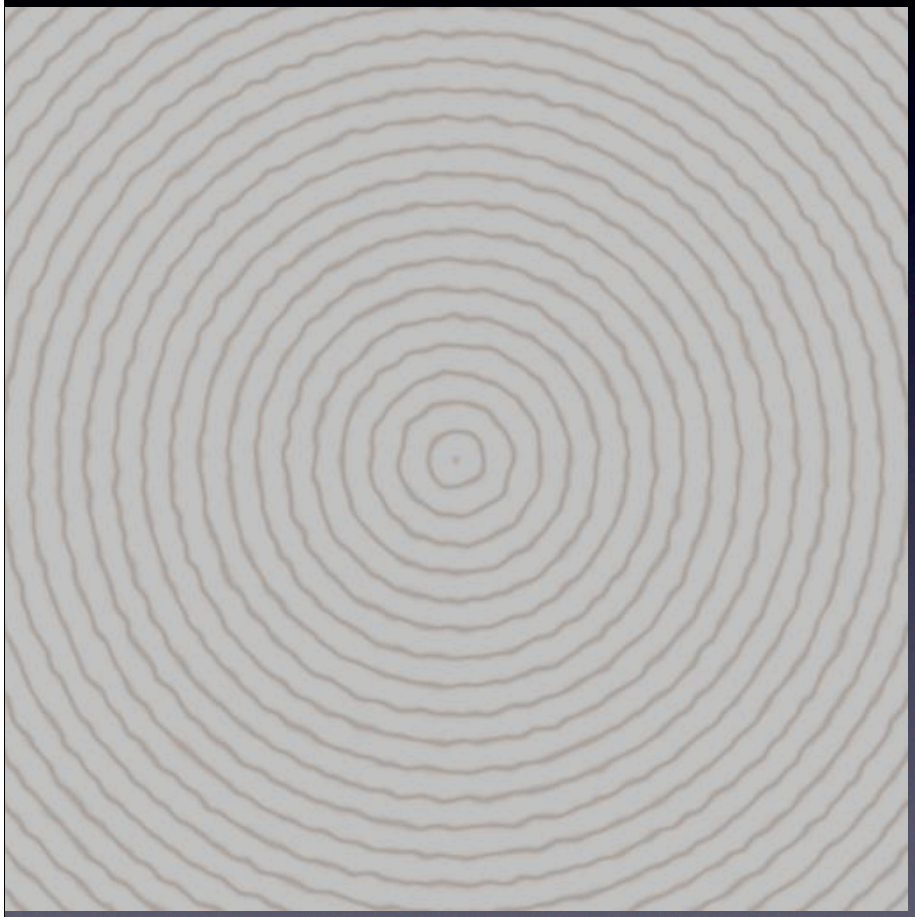
- Dark rings are a little further apart

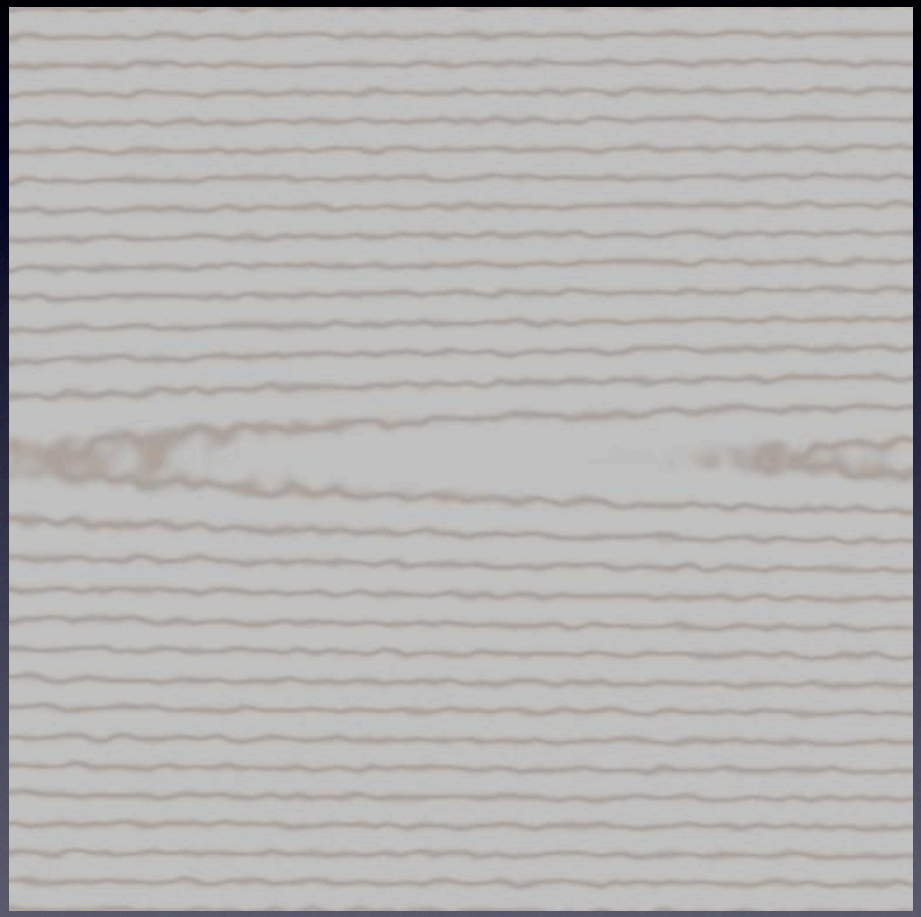


cos(radius)^8

Wood texture

- Rings aren't perfect

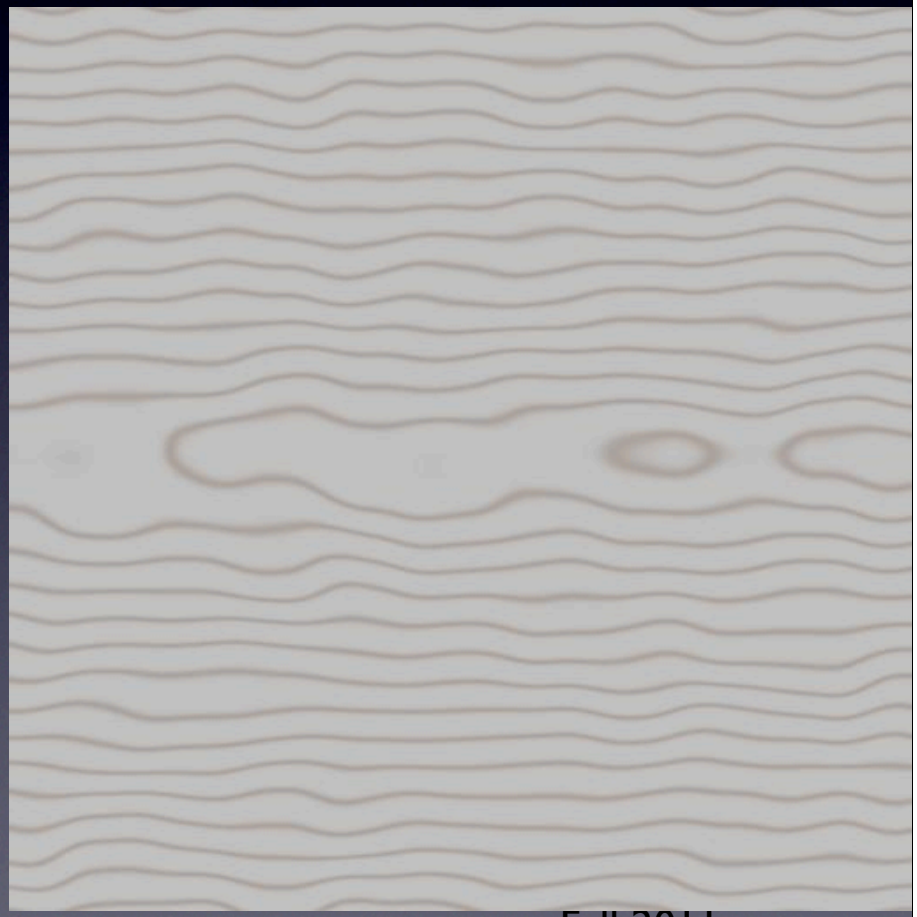




cos(R+noise(P))^8

Wood texture

- Large imperfections make the wood "curly"

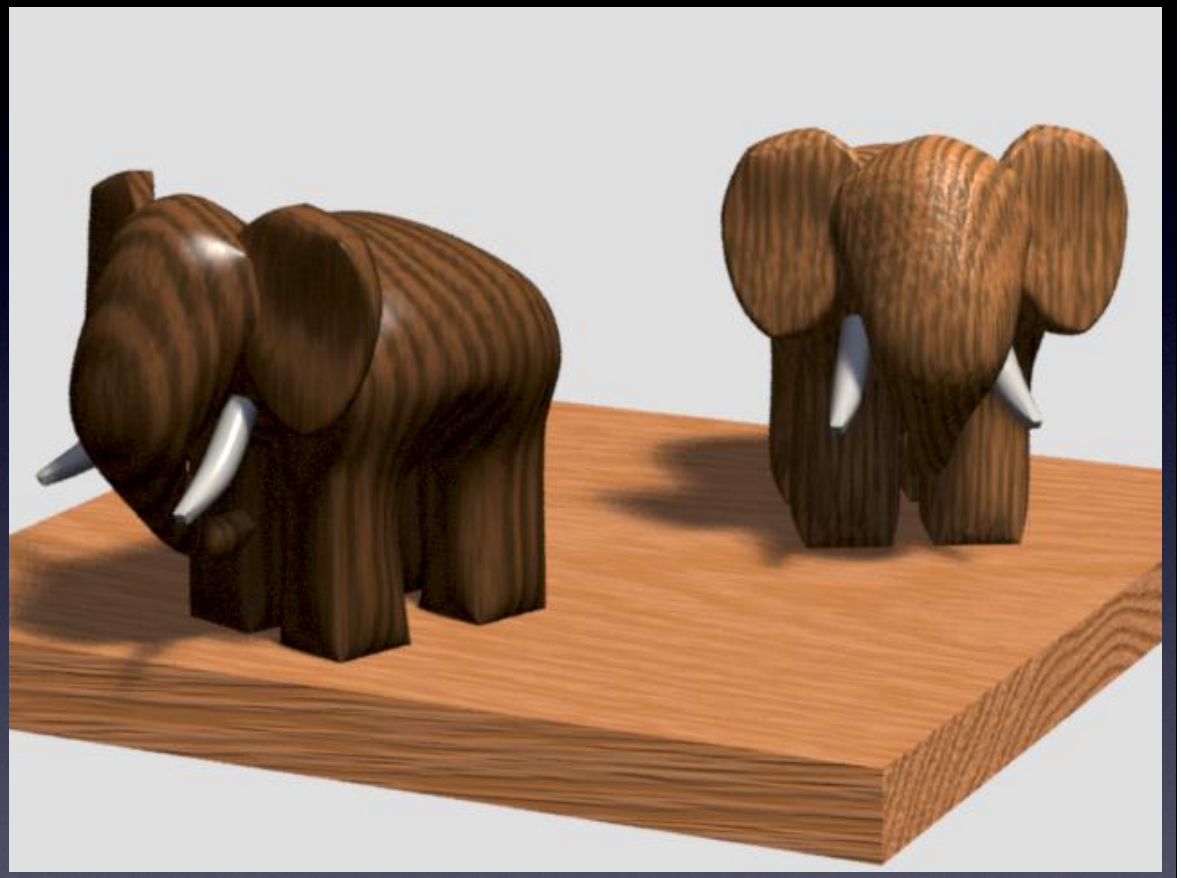


Other improvements

- Modulate center based on noise
- Model finer grain structure of wood grain
- Model rays

Advanced wood

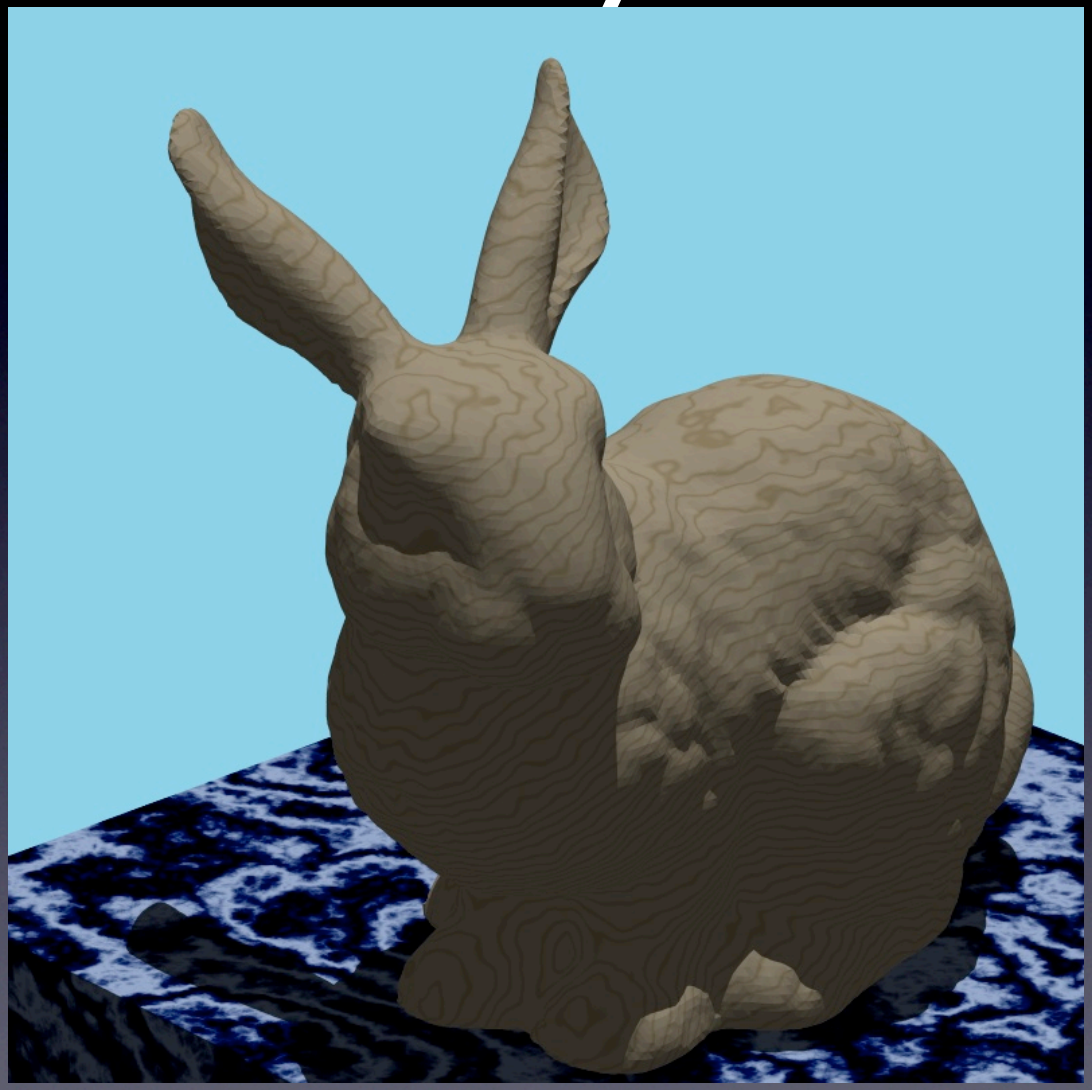
- Render grain
- Better transitions
- Shader from Advanced Renderman has 11 parameters



Advanced Renderman

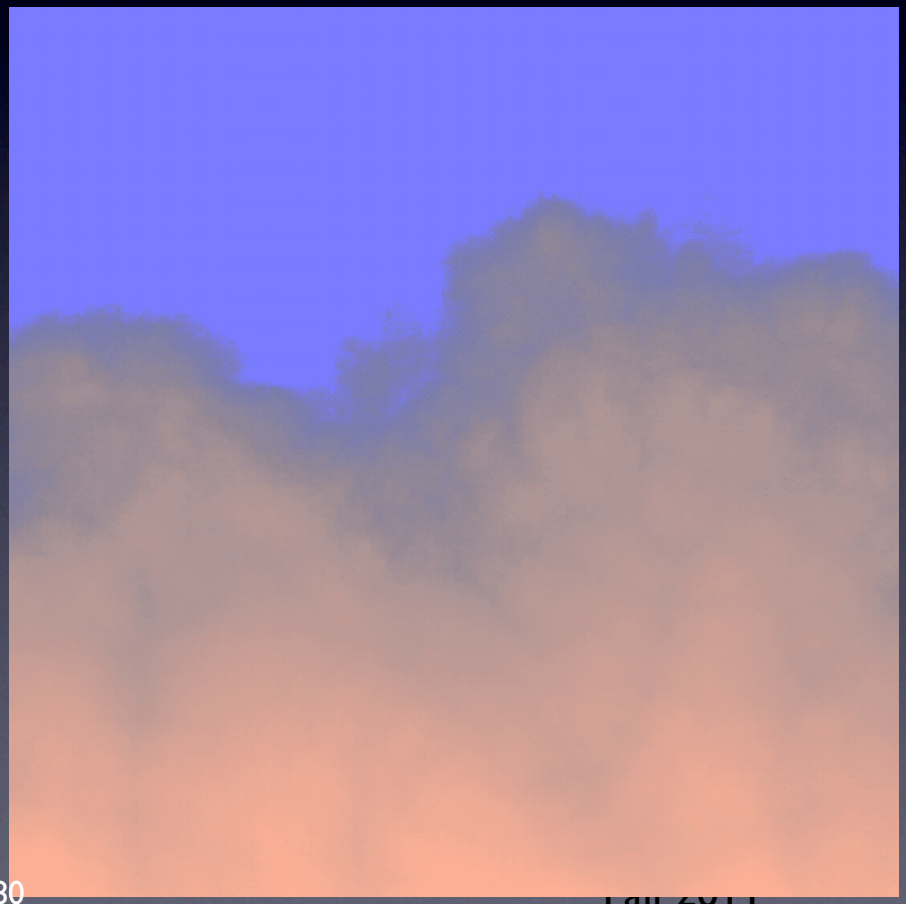
Wooden bunny

- Marble base and simple wood texture on bunny



Other noise tricks

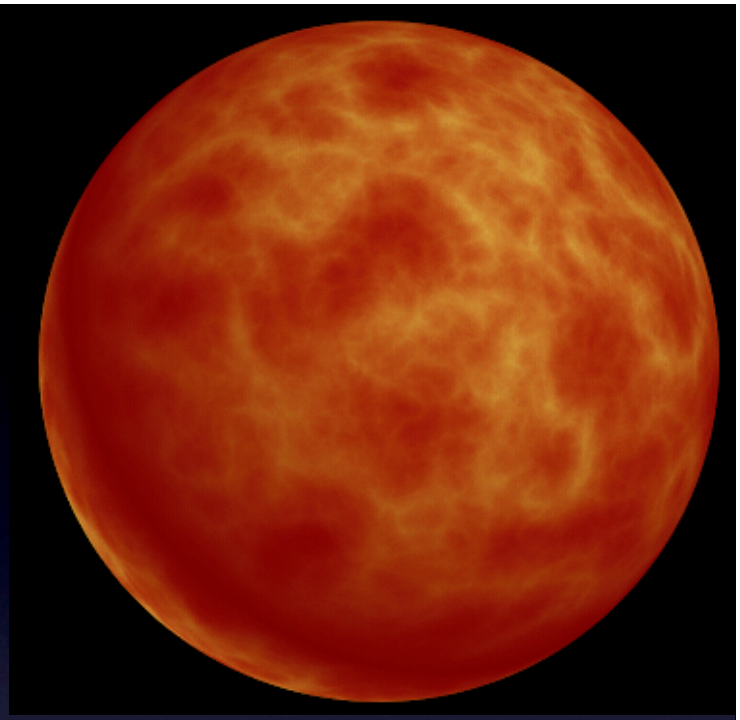
- Embed noise in higher dimension and animate (2D ->3D, 3D->4D)
  - Fire burning
  - Clouds evolving

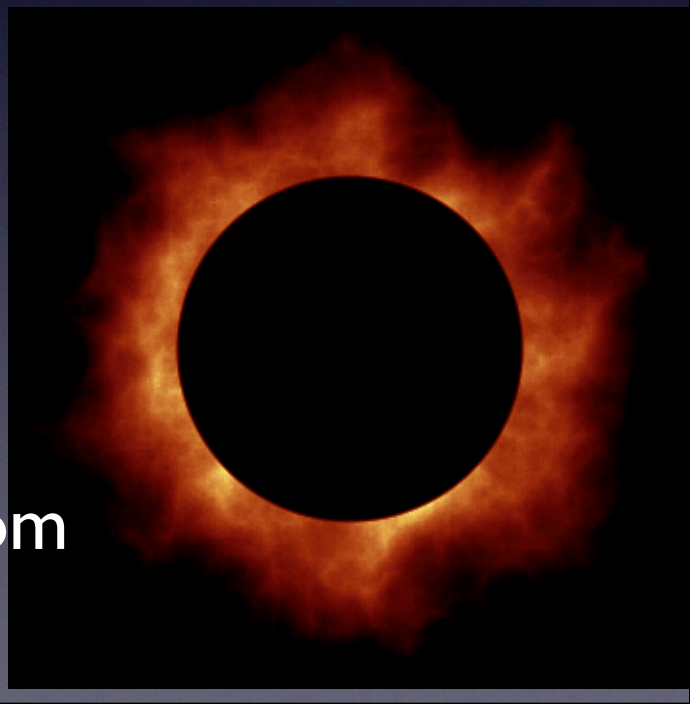


noisemachine.com

Other procedural textures

- Granite
- Stucco
- Dented/Eroded
- Flames
- Clouds
- Many more - use your imagination!

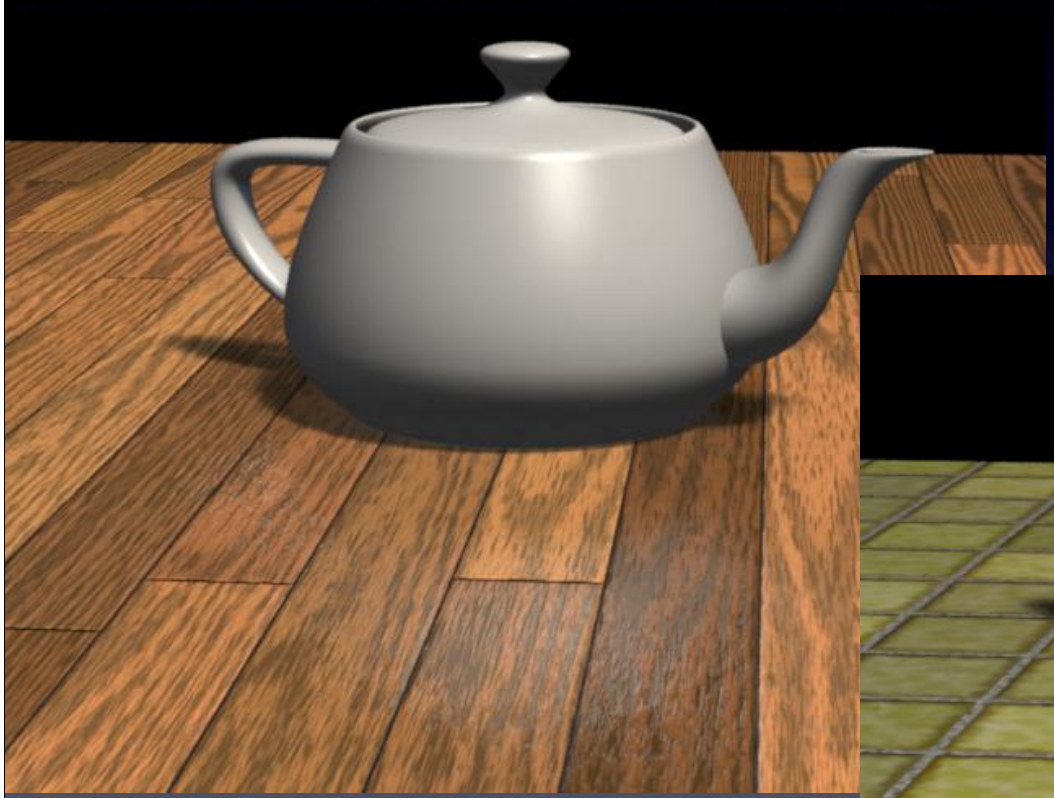


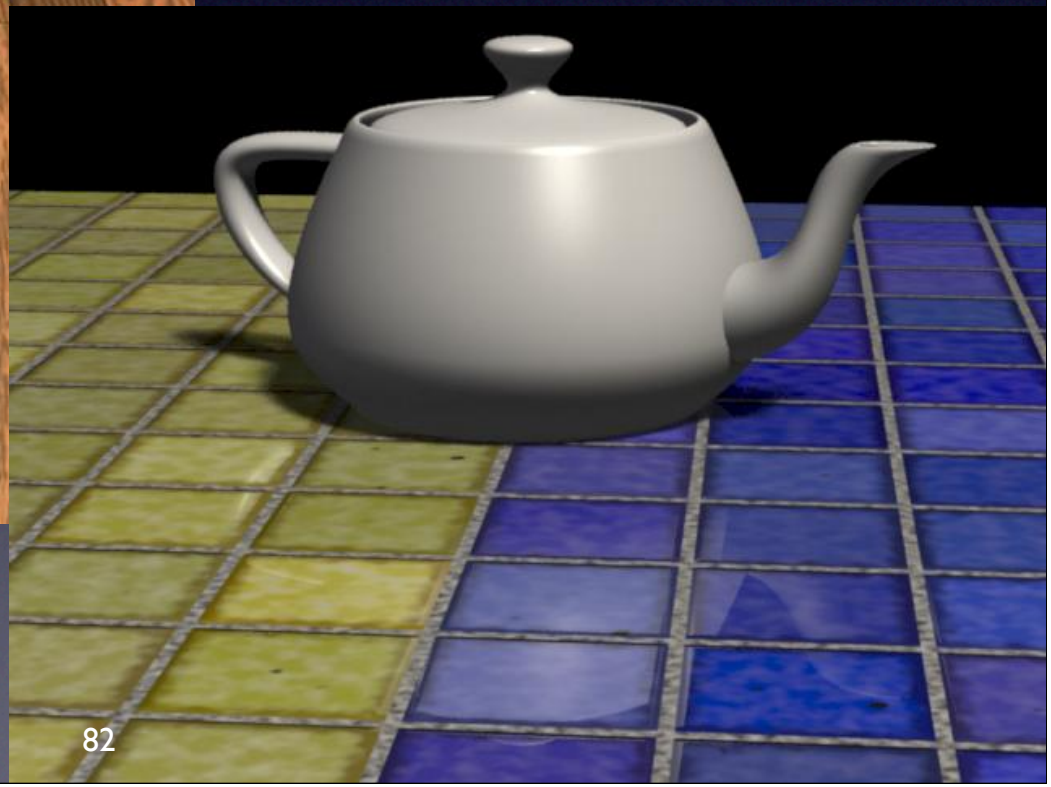


noisemachine.com

Combinations of shaders

- Combine shaders to get even better results





Recipe for success

- Add method to Primitive based class to compute texture coordinates:
- void computeUVW(Vector& uvw, const RenderContext& context const Ray& ray, const HitRecord& hit) const;
- Default implementation sets uvw as hit position:
- uvw = (ray.origin + ray.direction*t)

Polar Coordinates

- Used for left two spheres (with maps)
- Create subclass of Sphere (SpherePolar)
- Override computeUVW method to compute polar coordinates
- In constructor:

$$\vec{Z} = \overrightarrow{poleAxis} \text{ (4th argument)}$$

$$\vec{X} = \overrightarrow{primeMeridianAxis} \text{ (5th argument)}$$

$$\vec{Y} = \vec{Z} \times \vec{X}$$

normalize $\vec{\mathrm{X}}$, $\vec{\mathrm{Y}}$, and $\vec{\mathrm{Z}}$

Polar coordinates

$$\overrightarrow{P} = \overrightarrow{O} + t\overrightarrow{V}$$

$$\overrightarrow{P'} = \overrightarrow{P} - \overrightarrow{C}$$

$$x = \overrightarrow{P'} \cdot \overrightarrow{X}, y = \overrightarrow{P'} \cdot \overrightarrow{Y}, z = \overrightarrow{P'} \cdot \overrightarrow{Z}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \operatorname{atan2}(y, x)$$

$$u = \frac{\theta}{2\pi}$$

$$if\,(u < 0)u+ = 1$$

$$\phi = \arccos\left(\frac{z}{R}\right)$$

$$v = 1 - \frac{\phi}{\pi}$$

$$uvw = Vector(u, v, R)$$

Checkerboard texture

Checkerboard a subclass of Material

In constructor:

Given: $\overrightarrow{C}$ (center of checkerboard), $\overrightarrow{V_1}, \overrightarrow{V_2}$ (checker axis directions)

$\overrightarrow{\mathrm{V}_1}$ should be vector to first checker vertex

We want to switch checker vertices at integers 1,2,...

$$\therefore \overrightarrow{V_1} \cdot \overrightarrow{V_1'} = 1$$

$$\overrightarrow{V_1'} = \frac{\overrightarrow{V_1}}{\left\|\overrightarrow{V_1}\right\|^2}, \overrightarrow{V_2'} = \frac{\overrightarrow{V_2}}{\left\|\overrightarrow{V_2}\right\|^2}$$

Checkerboard texture

Call computeUVW on hit primitive: $\overrightarrow{\text{uvw}}$

$$i1 = floor\left(\overrightarrow{uvw} \cdot \overrightarrow{V_1'}\right)$$

$$i2 = floor\left(\overrightarrow{uvw} \cdot \overrightarrow{V_2'}\right)$$

$$which\_matl = (i1 + i2)\%2$$

$$if\,(which\_matl == 0)$$

```
  matl0->shade(....)
else
  matl1->shade(...)
```

Checkerboard optimization

- Put materials in a two-element array, indexed by which_matl
- Beware (-1)%2 = -1
- One solution: make a three-element array, indexed by which_matl+1
- matls[which_matl+1]->shade(…)

Phong Marble Material

Compute $\overrightarrow{uvw}$ using hit primitive

$$\vec{T} = \overrightarrow{uvw} * scale * fscale$$

$$value = \cos\left(u * scale + tscale * turbulenceAbs(octaves, \vec{T}, lacunarity, gain)\right)$$

$$value = value * 0.5 + 0.5$$

$$color = c_1(1 - value) + c_2 value$$

Use color in phong lighting calculations

Parameters:

scale = spatial magnification

fscale = frequency magnfication of turbulence (smaller:coarser structure)

tscale = distortion due to marble (0: pure sinusoid, large: more veins)

octaves = number of frequencies in fractal turbulence

lacunarity = frequency multiplication

gain = decay of higher frequencies in fractal turbulence

Phong Image Material

Compute $\overrightarrow{uvw}$ using hit primitive

$x = u * xres$

$y = v * yres$

$ix = Floor(x), iy = Floor(y)$

$fx = x - ix, fy = y - iy$

Interpolate color from image(ix,iy) using fx,fy

Use color in phong shading calculations

Image wrapping

Need to interpolate at prime meridian:

Interpolation:

```
  image(x,y)(1-fx)(1-fy)
+image((x+1)%xres, y)(fx)(1-fy)
+image(x,(y+1)%yres)(1-fx)(fy)
+image((x+1)%xres,(y+1)%yres)(fx)(fy)
```

This still isn't quite right, but it works fine for this assignment

You probably won't notice the difference unless you use a smaller image

Floor

- Note that Floor(x) != (int)x
- Wrong for x<0
- Integer cast actually undefined for x<0
- Most machines round toward 0 (not what we want)

Correct Floor implementation:

```
inline int Floor(float d)
{
  if(d<0){
    int i = -static_cast<int>(-d);
    if(i == d)
      return i;
    else
      return i-1;
  } else {
    return static_cast<int>(d);
  }
}
```