Homework #7 Solutions:

1. **4-49C** It can be used for any kind of process of an ideal gas.

2. **4-54C** The energy required is \(mc_pT\), which will be the same in both cases. This is because the \(c_p\) of an ideal gas does not vary with pressure.

3. **4-61** Neon is compressed isothermally in a compressor. The specific volume and enthalpy changes are to be determined.

   **Assumptions** At specified conditions, neon behaves as an ideal gas.

   **Properties** The gas constant of neon is \(R = 0.4119 \text{ kJ/kg} \cdot \text{K}\) and the constant-pressure specific heat of neon is \(1.0299 \text{ kJ/kg} \cdot \text{K}\) (Table A-2a).

   **Analysis** At the compressor inlet, the specific volume is
   \[
   v_1 = \frac{RT}{P_1} = \frac{(0.4119 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273 \text{ K})}{100 \text{ kPa}} = 1.207 \text{ m}^3/\text{kg}
   \]

   Similarly, at the compressor exit,
   \[
   v_2 = \frac{RT}{P_2} = \frac{(0.4119 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(20 + 273 \text{ K})}{500 \text{ kPa}} = 0.2414 \text{ m}^3/\text{kg}
   \]

   The change in the specific volume caused by the compressor is
   \[
   \Delta v = v_2 - v_1 = 0.2414 - 1.207 = -0.966 \text{ m}^3/\text{kg}
   \]

   Since the process is isothermal,
   \[
   \Delta h = c_p\Delta T = 0 \text{ kJ/kg}
   \]

4. **4-66** A resistance heater is to raise the air temperature in the room from 7 to 23°C within 15 min. The required power rating of the resistance heater is to be determined.

   **Assumptions** 1 Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. 2 The kinetic and potential energy changes are negligible, \(\Delta ke \Delta pe = 0\). 3 Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications. 4 Heat losses from the room are negligible. 5 The room is air-tight so that no air leaks in and out during the process.

   **Properties** The gas constant of air is \(R = 0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}\) (Table A-1). Also, \(c_v = 0.718 \text{ kJ/kg} \cdot \text{K}\) for air at room temperature (Table A-2).

   **Analysis** We take the air in the room to be the system. This is a closed system since no mass crosses the system boundary. The energy balance for this stationary constant-volume closed system can be expressed as
   \[
   \Delta E_{\text{in}} = \Delta E_{\text{out}} + \Delta E_{\text{system}}
   \]

   where \(\Delta E_{\text{in}}\) is the net energy transfer by heat, work, and mass, \(\Delta E_{\text{out}}\) is the change in internal, kinetic, potential, etc. energies, and \(\Delta E_{\text{system}}\) is the change in internal, kinetic, potential, etc. energies. The energy balance is
   \[
   W_{\text{in}} = \Delta U + mc_v\Delta v (T_2 - T_1) \quad \text{(since } Q = KE = PE = 0)\]
The mass of air is
\[ V = 4 \times 5 \times 6 = 120 \, \text{m}^3 \]
\[ m = \frac{P_1 V}{RT_1} = \frac{(100 \, \text{kPa})(120 \, \text{m}^3)}{(0.287 \, \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(280 \, \text{K})} = 149.3 \, \text{kg} \]
Substituting, the power rating of the heater becomes
\[ \dot{W}_{e,\text{in}} = \frac{(149.3 \, \text{kg})(0.718 \, \text{kJ/kg} \cdot \text{C})(23 \, \text{C})}{15 \times 60 \, \text{s}} = 1.91 \, \text{kW} \]

**Discussion** In practice, the pressure in the room will remain constant during this process rather than the volume, and some air will leak out as the air expands. As a result, the air in the room will undergo a constant pressure expansion process. Therefore, it is more proper to be conservative and to use \( \dot{H} \) instead of using \( \dot{U} \) in heating and air-conditioning applications.

5. **4-78** A cylinder equipped with a set of stops for the piston to rest on is initially filled with helium gas at a specified state. The amount of heat that must be transferred to raise the piston is to be determined.

**Assumptions**
1. Helium is an ideal gas with constant specific heats.
2. The kinetic and potential energy changes are negligible.
3. There are no work interactions involved.
4. The thermal energy stored in the cylinder itself is negligible.

**Properties** The specific heat of helium at room temperature is \( c_v = 3.1156 \, \text{kJ/kg} \cdot \text{K} \) (Table A-2).

**Analysis** We take the helium gas in the cylinder as the system. This is a closed system since no mass crosses the boundary of the system. The energy balance for this constant volume closed system can be expressed as

\[ E_{\text{in}} - E_{\text{out}} = E_{\text{system}} \]
\[ Q_{\text{in}} = m(u_2 - u_1) \]
\[ Q_{\text{in}} = m(u_2 - u_1) = mc_v(T_2 - T_1) \]

The final temperature of helium can be determined from the ideal gas relation to be
\[ \frac{P_1 V}{T_1} = \frac{P_2 V}{T_2} \Rightarrow T_2 = \frac{P_2 \, T_1}{P_1} = \frac{500 \, \text{kPa}}{100 \, \text{kPa}} (298 \, \text{K}) = 1490 \, \text{K} \]
Substituting into the energy balance relation gives
\[ Q_{\text{in}} = (0.5 \, \text{kg})(3.1156 \, \text{kJ/kg} \cdot \text{K})(1490 - 298) \text{K} = 1857 \, \text{kJ} \]

6. **4-90** Stainless steel ball bearings leaving the oven at a specified uniform temperature at a specified rate are exposed to air and are cooled before they are dropped into the water
for quenching. The rate of heat transfer from the ball bearing to the air is to be determined.

**Assumptions**
1. The thermal properties of the bearing balls are constant.
2. The kinetic and potential energy changes of the balls are negligible.
3. The balls are at a uniform temperature at the end of the process.

**Properties**
The density and specific heat of the ball bearings are given to be $\rho = 8085$ kg/m$^3$ and $c_p = 0.480$ kJ/kg°C.

**Analysis**
We take a single bearing ball as the system. The energy balance for this closed system can be expressed as

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{system}$$

where $\dot{E}_{in}$ is the energy input, $\dot{E}_{out}$ is the energy output, and $\dot{E}_{system}$ is the change in internal, kinetic, potential, etc. energies.

$$\dot{Q}_{out} = \dot{U}_{ball} = m (u_2 - u_1)$$

$$\dot{Q}_{out} = mc(T_1 - T_2)$$

The total amount of heat transfer from a ball is

$$m = \frac{\frac{4}{3} \pi D^3}{6} = \frac{(8085 \text{ kg/m}^3) \frac{(0.012 \text{ m})^3}{6}}{6} = 0.007315 \text{ kg}$$

$$\dot{Q}_{out} = mc(T_1 - T_2) = (0.007315 \text{ kg})(0.480 \text{ kJ/kg°C})(900 - 850)\text{°C} = 0.1756 \text{ kJ/ball}$$

Then the rate of heat transfer from the balls to the air becomes

$$\dot{Q}_{total} = n_{ball} \dot{Q}_{out (per \ ball)} = (800 \text{ balls/min})(0.1756 \text{ kJ/ball}) = 140.5 \text{ kJ/min} = 2.34 \text{ kW}$$

Therefore, heat is lost to the air at a rate of 2.34 kW.