

LES of Turbulent Flows: Lecture 13

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Classes of LES Models (for τ_{ij})

1. Eddy-Viscosity Models:

- The largest and most commonly used class of SGS models
- The general idea is that turbulent diffusion at SGSs (i.e. how SGS remove energy) is analogous to molecular diffusion. It is very similar in form to K-theory for Reynolds stresses.
- The deviatoric part of τ_{ij} is modeled as:

$$\tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij} = -2\nu_T \tilde{S}_{ij}$$

where ν_T is the SGS eddy-viscosity and $\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)$

Within this class of models we have many different ways of determining ν_T

- a) Smagorinsky model (Smagorinsky, Mon. Weath. Rev., 1963)
- b) “Kolmogorov” eddy-viscosity (Wong and Lilly, Phys. of Fluids, 1994)
- c) Two-point closure models (based on Kraichnan’s (JAS, 1974) spectral eddy-viscosity model and developed by Lesieur (see Lesieur et al., 2005)
- d) One-equation models that use the filtered KE equation (see Lecture 6 for the KE equation and Deardorff (BLM, 1980) for an early application of the model)

Classes of LES Models (for τ_{ij})

2. Similarity Models:

- Based on the idea that the most active SGS are those close to the cutoff wavenumber (or scale) these models were first introduced by Bardina et al. (AIAA, 1980). A subclass under this type of model is the nonlinear model. Many times these models are paired with an eddy-viscosity model.

3. Stochastic SGS models:

- In these models a random (stochastic) component is incorporated into the SGS model. Usually the nonlinear term is combined with an eddy-viscosity model in a similar manner to the similarity models (Mason and Thomson, JFM, 1992).

4. Subgrid Velocity Reconstruction Models:

- These models seek to approximate the SGS stress through a direct reconstruction of the SGS velocity or scalar fields. Two examples are:
 - fractal models (Scotti and Meneveau, Physica D, 1999)
 - Linear-eddy and ODT models (Kerstein, Comb. Sci. Tech, 1988)

5. Dynamic Models:

- Actually more of a procedure that can be applied with any base model, first developed by Germano (Phys of Fluids, 1991).
- One of the biggest and most influential ideas in LES SGS modeling.

Testing SGS Models

How do we test SGS models?

- ***a posteriori*** testing (term can be credited to Piomelli et al., Phys of Fluids, 1988)
 - Run “full” simulations with a particular SGS model and compare the results (statistically) to DNS, experiments and turbulence theory.
 - A “complete” test of the model (including dynamics and feedback) but it has the disadvantage of including numerics and **we can’t gain insight into the physics of SGSs.**
- ***a priori*** testing (term can be credited to Piomelli et al., Phys of Fluids, 1988)
 - Use DNS or high resolution experimental data to test SGS models “offline”
 - **Goal:** directly compare $\tau_{ij}^{\Delta}(\vec{x}, t)$ with $\tau_{ij}^{\Delta, M}(\vec{x}, t)$ by:
 - Filter DNS (or experimental) data at Δ and compute the exact $\tau_{ij}^{\Delta}(\vec{x}, t)$ and other relevant parameters (e.g., Π)
 - Use the filtered u, v and w to compute $\tau_{ij}^{\Delta, M}(\vec{x}, t)$ from the model (and stats) and compare with above results (this can also be used to calculate unknown model coefficients)
 - Allows us to look specifically at how the model reproduces SGS properties and the physics associated with those properties
 - Drawback, it doesn’t include dynamic feedback and numerics!

Eddy viscosity Models

- **Eddy Viscosity Models:** (Guerts pg 225; Pope pg 587)

-Above, we noted that eddy-viscosity models are of the form:

momentum: $\tau_{ij} = -2\nu_T \tilde{S}_{ij}$

← filtered strain rate

↙ eddy-viscosity

scalars: $q_i = -D_T \frac{\partial \tilde{\theta}}{\partial x_i}$ where $D_T = \frac{\nu_T}{Pr_{sgs}}$

↙ SGS Prandtl number

↙ eddy-diffusivity

-This is the LES equivalent to 1st order RANS closure (k-theory or gradient transport theory) and is an analogy to molecular viscosity (see Pope Ch. 10 for a review)

-Turbulent fluxes are assumed to be proportional to the local velocity or scalar gradients

-In LES this is the assumption that stress is proportional to strain: $\tau_{ij} \sim \tilde{S}_{ij}$

-The SGS eddy-viscosity ν_T must still be parameterized.

Eddy viscosity Models

-Dimensionally =>

$$\nu_T = \left[\frac{L^2}{T} \right]$$

-In almost all SGS eddy-viscosity models $\nu_T \sim u^* l^*$
velocity scale length scale

-Different models use different u^* and l^*

-Recall the filtered N-S equations:

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i$$

-Note, for an eddy-viscosity model we can write the viscous and SGS stress terms as:

$$\frac{1}{Re} \frac{\partial}{\partial x_j} \tilde{S}_{ij} \quad \text{and} \quad \tau_{ij} = -2\nu_T \tilde{S}_{ij}$$

We can combine these two and come up with a new (approximate) viscous term:

$$\frac{\partial}{\partial x_j} \left[\left(\nu_T + \frac{1}{Re} \right) \tilde{S}_{ij} \right]$$

What does the model do? We can see it effectively lowers the Reynolds number of the flow and for high Re (when $1/Re \Rightarrow 0$), it provides all of the energy dissipation.