1. (20 pts) Define the term ‘critical insulation radius’ both with words and with symbols. Explain what happens if $r_i < r_{cr}$ as well as what happens when $r_i > r_{cr}$. Also, compare the qualitative effects of the critical insulation radius for radial systems (cylinders and spheres) with those of plane walls.

2.* (10 pts) A hollow aluminum sphere, with an electrical heater in the center, is used in tests to determine the thermal conductivity of insulating materials. The inner and outer radii of the sphere are 0.15 and 0.18 m, respectively, and testing is done under steady-state conditions with the inner surface of the aluminum maintained at 250°C. In a particular test, a spherical shell of insulation is cast on the outer surface of the sphere to a thickness of 0.12 m. The system is in a room for which the air temperature is 20°C and the convection coefficient at the outer surface of the insulation is 30 W/m²·K. If 80 W are dissipated by the heater under steady-state conditions, what is the thermal conductivity of the insulation?

3.* (10 pts) The air inside a chamber at $T_{∞,i} = 50°C$ is heated convectively with $h_i = 20$ W/m²·K by a 200-mm-thick wall having a thermal conductivity of 4 W/m·K and uniform heat generation of 1000 W/m³. To prevent any heat generated within the wall from being lost to the outside of the chamber at $T_{∞,o} = 25°C$ with $h_o = 5$ W/m²·K, a very thin strip heater is placed on the outer wall to provide a uniform heat flux $q_o''$.

   (a) Sketch the temperature distribution in the wall on $T$-$x$ coordinates for the condition where no heat generated within the wall is lost to the outside of the chamber.
   
   (b) What are the temperatures at the wall boundaries, $T(0)$ and $T(L)$, for the conditions of part (a)?
   
   (c) Determine the value of $q_o''$ that must be supplied by the strip heater so that all heat generated within the wall is transferred to the inside of the chamber.
   
   (d) If the heat generation in the wall were switched off while the heat flux to the strip heater remained constant, what would be the steady state temperature, $T(0)$, of the outer wall surface?

* Solutions for these problems are available on the course website: www.chen3453.com
4.* (10 pts) A long, circular aluminum rod is attached at one end to a heated wall and transfers heat by conduction to a cold fluid.

(a) If the diameter of the rod is tripled, by how much would the rate of heat removal change?

(b) If a copper rod of the same diameter is used in place of the aluminum, by how much would the rate of heat removal change?

5.* (10 pts) A very long rod of 5-mm diameter and uniform thermal conductivity \( k = 25 \text{ W/m·K} \) is subjected to a heat treatment process. The center, 30-mm-long portion of the rod within the induction heating coil experiences uniform volumetric heat generation of \( 7.5 \times 10^6 \text{ W/m}^3 \).

The unheated portions of the rod, which protrude from the heating coil on either side, experience convection with the ambient air at \( T_\infty = 20^\circ \text{C} \) and \( h = 10 \text{ W/m}^2\cdot\text{K} \). Assume that there is no convection from the surface of the rod within the coil.

(a) Calculate the steady-state temperature \( T_o \) of the rod at the midpoint of the heated portion of the coil.

(b) Calculate the temperature of the rod \( T_b \) at the edge of the heated portion.

6. (20 pts) A 5 mm diameter copper wire (resistivity \( \rho = 1.72 \times 10^{-8} \Omega\cdot\text{m} \)) is insulated with a 100 mm thick layer of flexible polymer (\( k = 0.24 \text{ W/m·K} \)) which decomposes at temperatures greater than 55°C. If the outer surface of the insulating layer is kept at 20°C, what is the maximum amount of current that can pass through the wire?

Hint: For electric heating, power (Watts) = \( I^2R \) where \( I \) = current and \( R = \rho L/A \) where \( L \) is length and \( A \) is cross sectional area

7. (20 pts) A cylindrical stainless steel vessel (\( k = 15 \text{ W/m·K} \)) with an inside diameter of 1 meter and 0.1 meter thick walls is full of radioactive material (\( k = 80 \text{ W/m·K} \)) which generates energy at a rate of \( 2 \times 10^5 \text{ W/m}^3 \). The vessel is submerged in a bath of water, which is maintained at 25°C. The convective heat transfer coefficient between the vessel and water is 1000 W/m²·K. The ends of the cylindrical vessel are capped and very well insulated so that heat transfer through the ends is negligible. If the system is at steady state, determine the temperatures (a) at the outer surface of the vessel wall, (b) at the inner surface of the vessel wall and (c) at the center of the radioactive material.

EXTRA CREDIT (20 pts): Staple a copy of your current, updated resume to the back of your assignment.

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