Transient Conduction: Lumped Capacitance

Reminders…

• Homework #5 due Friday next week

• Midterm #1 coming up Wed. October 1
  – Covers chapters 1, 2, 3, 4, 5

• Engineering career fair Sept 23
  – Professionalism!!

• Bethany not available for office hours today
Some Thought Exercises

• Imagine an apple pie coming out of the oven at 200°C and being set on a window sill at 20°C. How long will it take to cool?

• Imagine a 5-cm steel ball coming out of a furnace at 530°C and being placed in a room at 30°C. How long will it take to cool?

The Biot Number

\[
\text{Bi} = \frac{hL}{k}
\]

• If Bi < 0.1 then the lumped capacitance approach can be used
  – Eq. 5.5 to find time to reach a given T
  – Eq. 5.6 to find T after a given time
  – Eq. 5.8a to find total heat gain (loss) for given time

• L depends on geometry
  – General approach is \( L = \frac{V}{A_s} \)
    • \( L/2 \) for wall with both sides exposed
    • \( r_o/2 \) for long cylinder
    • \( r_o/3 \) for sphere
  – Conservative approach is to use the maximum length
    • \( L/2 \) for wall exposed on both sides
    • \( r_o \) for cylinder or sphere (preferred to use this)
Time to Reach Temperature

\[ t = \frac{\rho Vc}{hA_s} \ln \frac{\theta_i}{\theta} \]  \hspace{1cm} \text{Eq. 5.5}

\[ \theta_i = T_i - T_\infty \]
\[ = \text{difference between initial object temperature and bulk fluid temperature} \]

\[ \theta = T - T_\infty \]
\[ = \text{difference between object temperature and bulk fluid temperature at time } t \]

Lumped Capacitance Equations

- Time to reach specified temperature (5.5):
  \[ t = \frac{\rho Vc}{hA_s} \ln \frac{\theta_i}{\theta} \]

- Temperature after specified time (5.6):
  \[ \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp \left[ -\left( \frac{hA_s}{\rho Vc} \right) t \right] \]

- Thermal time constant (5.7):
  \[ \tau_t = \left( \frac{1}{hA_s} \right) (\rho Vc) = R_i C_t \]

- Heat transferred during heating (5.8a):
  \[ Q = (\rho Vc) \theta_i \left[ 1 - \exp \left( -\frac{t}{\tau_t} \right) \right] \]
Example - Steel Ball

• How long does it take for the 5-cm steel ball to cool to 30°C?

• What is the temperature of the ball after 30 minutes?

• What was the total energy loss (Joules) from the ball?

Other relations

• Temperature after a given time
  \[
  \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp \left[ - \left( \frac{hA_s}{\rho Vc} \right) t \right]
  \]

• Thermal time constant
  \[
  \tau_t = \left( \frac{1}{hA_s} \right) \left( \rho Vc \right) = R_t C_t
  \]

• Heat transferred during heating or cooling
  \[
  Q = \left( \rho Vc \right) \theta_i \left[ 1 - \exp \left( - \frac{t}{\tau_t} \right) \right]
  \]
About that Pie…
(Can we use a lumped analysis approach?)

\[
\rho = 1000 \text{ kg/m}^3 \\
c = 4184 \text{ J/kg} \cdot \text{K} \\
k = 0.68 \text{ W/m} \cdot \text{K}
\]

Effect of Biot Number